Beyond Bird's Nested Arrays V

How about extending even beyond my Hierarchical Hyper-Nested Array Notation, which has a limit ordinal of $\theta(\Omega_\omega)$? Let me introduce to you an all-new 1-hyperseparator symbol – the black circle (●). Since the new symbol is a 1-hyperseparator (like the forward slash), it requires a minimum of one pair of square brackets around it, so the smallest separator containing ● is the [1 ● 2] separator.

The $\theta(\Omega_\omega)$ level separator

{a, b [1 ● 2] 2} = {a ‹0 ● 2› b}

= {a ‹b ‹b ‹ ... ‹b ‹b $/_b$ b› b› ... › b› b› b}

(with b pairs of angle brackets).

This is equivalent to

{a, b [1 [1 [... [1 [1 $/_b$ 1, 2] 2] ...] 2] 2] 2} (with b pairs of square brackets),

where the separator has level $\theta(\varepsilon(\Omega_{b-1}+\omega)) = \theta(\theta_{b-1}(1, \omega))$ (b ≥ 2).

Using the collapsing theta function (in single-argument form when the second argument is zero) for expressing ordinals beyond the epsilon numbers, I find that the more significant separators have ordinal levels as follows:-

[1 ● 2] has level $\theta(\Omega_\omega)$,
[2 ● 2] has level $\theta(\Omega_\omega)+1$,
[1 [1 / 2] 2 ● 2] has level $\theta(\Omega_\omega)+\varepsilon_0$,
[1 [1 ● 2] 2 ● 2] has level $\theta(\Omega_\omega)2$,
[1 [1 [1 ● 2] 2 ● 2] 2 ● 2] has level $\theta(\Omega_\omega)$^$\theta(\Omega_\omega)$,
[1 / 2 ● 2] has level $\varepsilon(\theta(\Omega_\omega)+1) = \theta(1, \theta(\Omega_\omega)+1)$,
[1 [1 / 2 $/_2$ 2] 2 ● 2] has level $\Gamma(\theta(\Omega_\omega)+1) = \theta(\Omega, \theta(\Omega_\omega)+1)$,
[1 [1 [1 $/_2$ 2 $/_3$ 2] 2] 2 ● 2] has level $\theta(\theta_1(\Omega_2), \theta(\Omega_\omega)+1)$,
[1 [1 [1 [1 $/_3$ 2 $/_4$ 2] 2] 2] 2 ● 2] has level $\theta(\theta_1(\Omega_3), \theta(\Omega_\omega)+1) = \theta(\theta_1(\theta_2(\Omega_3)), \theta(\Omega_\omega)+1)$,
[1 ● 3] has level $\theta(\theta_1(\Omega_\omega), 1)$
(limit ordinal of $\theta(\alpha, \theta(\Omega_\omega)+1) = \theta(\alpha, \theta(\theta_1(\Omega_\omega), 0)+1)$ as $\alpha \to \theta_1(\Omega_\omega)$),
[1 ● 4] has level $\theta(\theta_1(\Omega_\omega), 2)$,
[1 ● 1 [1 / 2] 2] has level $\theta(\theta_1(\Omega_\omega), \varepsilon_0)$,
[1 ● 1 [1 ● 2] 2] has level $\theta(\theta_1(\Omega_\omega), \theta(\Omega_\omega))$,
[1 ● 1 [1 ● 1 [1 ● 2] 2] 2] has level $\theta(\theta_1(\Omega_\omega), \theta(\theta_1(\Omega_\omega), \theta(\Omega_\omega)))$,
[1 ● 1 / 2] has level $\theta(\theta_1(\Omega_\omega)+1)$,
[1 ● 1 [2 $/_2$ 2] 2] has level $\theta(\theta_1(\Omega_\omega)+\omega)$,
[1 ● 1 [1 / 2 $/_2$ 2] 2] has level $\theta(\theta_1(\Omega_\omega)+\Omega)$,
[1 ● 1 [1 [1 $/_2$ 2 $/_3$ 2] 2] 2] has level $\theta(\theta_1(\Omega_\omega)+\theta_1(\Omega_2))$,
[1 ● 1 [1 [1 [1 $/_3$ 2 $/_4$ 2] 2] 2] 2] has level $\theta(\theta_1(\Omega_\omega)+\theta_1(\Omega_3))$,
[1 ● 1 ● 2] has level $\theta(\theta_1(\Omega_\omega)2)$,
[1 ● 1 ● 1 ● 2] has level $\theta(\theta_1(\Omega_\omega)3)$,
[1 ● 1 ● 1 ● ... ● 1 ● 2] (with n ● symbols) has level $\theta(\theta_1(\Omega_\omega)n)$.

With k ● symbols (k ≥ 1) and # representing the remainder of the array,

{a, b [1 ● 1 ● 1 ● ... ● 1 ● c #] 2} = {a ‹ 0 ● 1 ● 1 ● ... ● 1 ● c # › b}

= {a ‹ S ● S ● ... ● S ● T ● c-1 # › b},

where S = 'b / b / b / ... / b' (with b b's, explained on page 6),

T = 'b ‹b ‹b ‹ ... ‹b ‹b $/_b$ b› b› ... › b› b› b' (with b-1 pairs of angle brackets).

Introducing a new family of n-hyperseparator $\bullet_n$ symbols, with $\bullet = [1 \bullet_2 2]$ and the corresponding n-hyperseparator symbol $\bullet_n = [1 \bullet_{n+1} 2]$ in a similar manner to $/ = [1 /_2 2]$ and $/_n = [1 /_{n+1} 2]$ in Beyond Bird's Nested Arrays III, I find that

$[1\ [2 \bullet_2 2]\ 2]$ has level $\theta(\theta_1(\Omega_\omega)\omega)$,
$[1\ [1\ [1 \bullet 2]\ 2 \bullet_2 2]\ 2]$ has level $\theta(\theta_1(\Omega_\omega)\theta(\Omega_\omega))$,
$[1\ [1\ [1\ [1\ [1 \bullet 2]\ 2 \bullet_2 2]\ 2]\ 2 \bullet_2 2]\ 2]$ has level $\theta(\theta_1(\Omega_\omega)\theta(\theta_1(\Omega_\omega)\theta(\Omega_\omega)))$,
$[1\ [1 / 2 \bullet_2 2]\ 2]$ has level $\theta(\theta_1(\Omega_\omega)\Omega)$,
$[1\ [1 / 1 / 2 \bullet_2 2]\ 2]$ has level $\theta(\theta_1(\Omega_\omega)(\Omega^{\Omega}))$,
$[1\ [1\ [1 / 2 /_2 2]\ 2 \bullet_2 2]\ 2]$ has level $\theta(\theta_1(\Omega_\omega)(\Omega^{\Omega^{\Omega}}))$,
$[1\ [1\ [1 /_2 3]\ 2 \bullet_2 2]\ 2]$ has level $\theta(\theta_1(\Omega_\omega)\varepsilon_{\Omega+1})$,
$[1\ [1\ [1\ [1 /_2 2 /_3 2]\ 2]\ 2 \bullet_2 2]\ 2]$ has level $\theta(\theta_1(\Omega_\omega)\theta_1(\Omega_2))$,
$[1\ [1\ [1\ [1\ [1 /_3 2 /_4 2]\ 2]\ 2]\ 2 \bullet_2 2]\ 2]$ has level $\theta(\theta_1(\Omega_\omega)\theta_1(\Omega_3))$,
$[1\ [1 \bullet 2 \bullet_2 2]\ 2]$ has level $\theta(\theta_1(\Omega_\omega)^{2})$,
$[1\ [1 \bullet 1 \bullet 2 \bullet_2 2]\ 2]$ has level $\theta(\theta_1(\Omega_\omega)^{\theta_1(\Omega_\omega)})$,
$[1\ [1\ [1 \bullet 2 \bullet_2 2]\ 2 \bullet_2 2]\ 2]$ has level $\theta(\theta_1(\Omega_\omega)^{\theta_1(\Omega_\omega)^{\theta_1(\Omega_\omega)}})$,
$[1\ [1 /_2 2 \bullet_2 2]\ 2]$ has level $\theta(\varepsilon(\theta_1(\Omega_\omega)+1))$,
$[1\ [1\ [1 / 2 /_3 2]\ 2 \bullet_2 2]\ 2]$ has level $\theta(\theta_1(\Omega, \theta_1(\Omega_\omega)+1))$,
$[1\ [1\ [1 /_2 2 /_3 2]\ 2 \bullet_2 2]\ 2]$ has level $\theta(\theta_1(\Omega_2, \theta_1(\Omega_\omega)+1))$,
$[1\ [1\ [1\ [1 /_3 2 /_4 2]\ 2]\ 2 \bullet_2 2]\ 2]$ has level $\theta(\theta_1(\theta_2(\Omega_3), \theta_1(\Omega_\omega)+1))$,
$[1\ [1\ [1\ [1\ [1 /_4 2 /_5 2]\ 2]\ 2]\ 2 \bullet_2 2]\ 2]$ has level $\theta(\theta_1(\theta_2(\Omega_4), \theta_1(\Omega_\omega)+1))$,
$[1\ [1 \bullet_2 3]\ 2]$ has level $\theta(\theta_2(\Omega_\omega), 1) = \theta(\theta_1(\theta_2(\Omega_\omega), 1))$
($\theta_1(\theta_2(\Omega_\omega), 1)$ is the limit of $\theta_1(\alpha, \theta_1(\theta_2(\Omega_\omega))+1)$ as $\alpha \to \theta_2(\Omega_\omega)$),
$[1\ [1 \bullet_2 1 / 2]\ 2]$ has level $\theta(\theta_2(\Omega_\omega), \Omega)$,
$[1\ [1 \bullet_2 1 \bullet 2]\ 2]$ has level $\theta(\theta_2(\Omega_\omega), \theta_1(\Omega_\omega))$,
$[1\ [1 \bullet_2 1 /_2 2]\ 2]$ has level $\theta(\theta_2(\Omega_\omega)+1)$,
$[1\ [1 \bullet_2 1\ [1 / 2 /_3 2]\ 2]\ 2]$ has level $\theta(\theta_2(\Omega_\omega)+\Omega)$,
$[1\ [1 \bullet_2 1\ [1 /_2 2 /_3 2]\ 2]\ 2]$ has level $\theta(\theta_2(\Omega_\omega)+\Omega_2)$,
$[1\ [1 \bullet_2 1\ [1\ [1 /_3 2 /_4 2]\ 2]\ 2]\ 2]$ has level $\theta(\theta_2(\Omega_\omega)+\theta_2(\Omega_3))$,
$[1\ [1 \bullet_2 1 \bullet_2 2]\ 2]$ has level $\theta(\theta_2(\Omega_\omega)2)$,
$[1\ [1\ [2 \bullet_3 2]\ 2]\ 2]$ has level $\theta(\theta_2(\Omega_\omega)\omega)$,
$[1\ [1\ [1 / 2 \bullet_3 2]\ 2]\ 2]$ has level $\theta(\theta_2(\Omega_\omega)\Omega)$,
$[1\ [1\ [1 \bullet 2 \bullet_3 2]\ 2]\ 2]$ has level $\theta(\theta_2(\Omega_\omega)\theta_1(\Omega_\omega))$,
$[1\ [1\ [1 /_2 2 \bullet_3 2]\ 2]\ 2]$ has level $\theta(\theta_2(\Omega_\omega)\Omega_2)$,
$[1\ [1\ [1 \bullet_2 2 \bullet_3 2]\ 2]\ 2]$ has level $\theta(\theta_2(\Omega_\omega)^{2})$,
$[1\ [1\ [1 \bullet_2 1 \bullet_2 2 \bullet_3 2]\ 2]\ 2]$ has level $\theta(\theta_2(\Omega_\omega)^{\theta_2(\Omega_\omega)})$,
$[1\ [1\ [1\ [1 \bullet_2 2 \bullet_3 2]\ 2 \bullet_3 2]\ 2]\ 2]$ has level $\theta(\theta_2(\Omega_\omega)^{\theta_2(\Omega_\omega)^{\theta_2(\Omega_\omega)}})$,
$[1\ [1\ [1 /_3 2 \bullet_3 2]\ 2]\ 2]$ has level $\theta(\varepsilon(\theta_2(\Omega_\omega)+1))$,
$[1\ [1\ [1\ [1 / 2 /_4 2]\ 2 \bullet_3 2]\ 2]\ 2]$ has level $\theta(\theta_2(\Omega, \theta_2(\Omega_\omega)+1))$,
$[1\ [1\ [1\ [1 /_2 2 /_4 2]\ 2 \bullet_3 2]\ 2]\ 2]$ has level $\theta(\theta_2(\Omega_2, \theta_2(\Omega_\omega)+1))$,
$[1\ [1\ [1\ [1 /_3 2 /_4 2]\ 2 \bullet_3 2]\ 2]\ 2]$ has level $\theta(\theta_2(\Omega_3, \theta_2(\Omega_\omega)+1))$,
$[1\ [1\ [1\ [1\ [1 /_4 2 /_5 2]\ 2]\ 2 \bullet_3 2]\ 2]\ 2]$ has level $\theta(\theta_2(\theta_3(\Omega_4), \theta_2(\Omega_\omega)+1))$,
$[1\ [1\ [1 \bullet_3 3]\ 2]\ 2]$ has level $\theta(\theta_3(\Omega_\omega), 1) = \theta(\theta_2(\theta_3(\Omega_\omega), 1))$
($\theta_2(\theta_3(\Omega_\omega), 1)$ is the limit of $\theta_2(\alpha, \theta_2(\theta_3(\Omega_\omega))+1)$ as $\alpha \to \theta_3(\Omega_\omega)$),
$[1\ [1\ [1 \bullet_3 1 / 2]\ 2]\ 2]$ has level $\theta(\theta_3(\Omega_\omega), \Omega)$,
$[1\ [1\ [1 \bullet_3 1 /_2 2]\ 2]\ 2]$ has level $\theta(\theta_3(\Omega_\omega), \Omega_2)$,
$[1\ [1\ [1 \bullet_3 1 /_3 2]\ 2]\ 2]$ has level $\theta(\theta_3(\Omega_\omega)+1)$,
$[1\ [1\ [1 \bullet_3 1 \bullet_3 2]\ 2]\ 2]$ has level $\theta(\theta_3(\Omega_\omega)2)$,
$[1\ [1\ [1\ [2 \bullet_4 2]\ 2]\ 2]\ 2]$ has level $\theta(\theta_3(\Omega_\omega)\omega)$,
$[1\ [1\ [1\ [1 \bullet_3 2 \bullet_4 2]\ 2]\ 2]\ 2]$ has level $\theta(\theta_3(\Omega_\omega)^{2})$,

[1 [1 [1 [1 $/_4$ 2 $\bullet_4$ 2] 2] 2] 2] has level $\theta(\varepsilon(\theta_3(\Omega_\omega)+1))$,
[1 [1 [1 [1 $\bullet_4$ 3] 2] 2] 2] has level $\theta(\theta_4(\Omega_\omega), 1)$,
[1 [1 [1 [1 [1 $\bullet_5$ 3] 2] 2] 2] 2] has level $\theta(\theta_5(\Omega_\omega), 1)$,
[1 [1 [1 [1 [1 [1 $\bullet_6$ 3] 2] 2] 2] 2] 2] has level $\theta(\theta_6(\Omega_\omega), 1)$.
The limit ordinal of $\theta(\theta_n(\Omega_\omega), 1)$ as $n \to \omega$ is $\theta(\Omega_\omega, 1)$.

At this stage, I need new symbols that come next in the sequence after / and $\bullet$. I now take /[1 $_n$] to be $/_n$ and /[2 $_n$] to be $\bullet_n$ – these are followed by /[3 $_n$], /[4 $_n$], /[5 $_n$] etc. (When n = 1, /[1] is / and /[2] is $\bullet$.) In general, when X is an array, the n-hyperseparator /[X $_n$] = [1 /[X $_{n+1}$] 2]. The subscript is omitted when it is 1, since this is the lowest value. I obtain the following separators:

[1 /[3] 2] has level $\theta(\Omega_\omega, 1)$,
[1 /[4] 2] has level $\theta(\Omega_\omega, 2)$ (achieved by replacing $\bullet_n$ and $\theta_n(\Omega_\omega)$ in the previous paragraph by /[3 $_n$] and $\theta_n(\Omega_\omega, 1)$ respectively),
[1 /[1, 2] 2] has level $\theta(\Omega_\omega, \omega)$,
[1 /[1, 1, 2] 2] has level $\theta(\Omega_\omega, \omega\hat{}2)$,
[1 /[1 [2] 2] 2] has level $\theta(\Omega_\omega, \omega\hat{}\omega)$,
[1 /[1 [1 / 2] 2] 2] has level $\theta(\Omega_\omega, \varepsilon_0)$,
[1 /[1 [1 $\bullet$ 2] 2] 2] has level $\theta(\Omega_\omega, \theta(\Omega_\omega))$,
[1 /[1 [1 /[1 [1 $\bullet$ 2] 2] 2] 2] 2] has level $\theta(\Omega_\omega, \theta(\Omega_\omega, \theta(\Omega_\omega)))$,
[1 /[1 / 2] 2] has level $\theta(\Omega_\omega, \Omega)$,
[1 /[1 / 3] 2] has level $\theta(\Omega_\omega, \Omega 2)$,
[1 /[1 / 1 / 2] 2] has level $\theta(\Omega_\omega, \Omega\hat{}2)$,
[1 /[1 [2 $/_2$ 2] 2] 2] has level $\theta(\Omega_\omega, \Omega\hat{}\omega)$,
[1 /[1 [1 / 2 $/_2$ 2] 2] 2] has level $\theta(\Omega_\omega, \Omega\hat{}\Omega)$,
[1 /[1 [1 $/_2$ 3] 2] 2] has level $\theta(\Omega_\omega, \varepsilon_{\Omega+1}) = \theta(\Omega_\omega, \theta_1(1))$,
[1 /[1 [1 $/_2$ 1 $/_2$ 2] 2] 2] has level $\theta(\Omega_\omega, \zeta_{\Omega+1}) = \theta(\Omega_\omega, \theta_1(2))$,
[1 /[1 [1 [2 $/_3$ 2] 2] 2] 2] has level $\theta(\Omega_\omega, \theta_1(\omega))$,
[1 /[1 [1 [1 / 2 $/_3$ 2] 2] 2] 2] has level $\theta(\Omega_\omega, \theta_1(\Omega))$,
[1 /[1 [1 [1 $/_2$ 2 $/_3$ 2] 2] 2] 2] has level $\theta(\Omega_\omega, \theta_1(\Omega_2))$,
[1 /[1 [1 [1 [1 $/_3$ 2 $/_4$ 2] 2] 2] 2] 2] has level $\theta(\Omega_\omega, \theta_1(\Omega_3))$,
[1 /[1 $\bullet$ 2] 2] has level $\theta(\Omega_\omega, \theta_1(\Omega_\omega))$,
[1 /[1 [1 $\bullet$ 2 $\bullet_2$ 2] 2] 2] has level $\theta(\Omega_\omega, \theta_1(\Omega_\omega)\hat{}\theta_1(\Omega_\omega))$,
[1 /[1 [1 $/_2$ 2 $\bullet_2$ 2] 2] 2] has level $\theta(\Omega_\omega, \varepsilon(\theta_1(\Omega_\omega)+1)) = \theta(\Omega_\omega, \theta_1(1, \theta_1(\Omega_\omega)+1))$,
[1 /[1 [1 $\bullet_2$ 3] 2] 2] has level $\theta(\Omega_\omega, \theta_1(\theta_2(\Omega_\omega), 1))$,
[1 /[1 [1 [1 $\bullet_3$ 3] 2] 2] 2] has level $\theta(\Omega_\omega, \theta_1(\theta_3(\Omega_\omega), 1))$,
[1 /[1 [1 [1 [1 $\bullet_4$ 3] 2] 2] 2] 2] has level $\theta(\Omega_\omega, \theta_1(\theta_4(\Omega_\omega), 1))$,
[1 /[1 /[3] 2] 2] has level $\theta(\Omega_\omega, \theta_1(\Omega_\omega, 1))$,
[1 /[1 /[1 / 2] 2] 2] has level $\theta(\Omega_\omega, \theta_1(\Omega_\omega, \Omega))$,
[1 /[1 /[1 $\bullet$ 2] 2] 2] has level $\theta(\Omega_\omega, \theta_1(\Omega_\omega, \theta_1(\Omega_\omega)))$,
[1 /[1 /[1 /[1 $\bullet$ 2] 2] 2] 2] has level $\theta(\Omega_\omega, \theta_1(\Omega_\omega, \theta_1(\Omega_\omega, \theta_1(\Omega_\omega))))$.
The limit ordinal of the /[ ] bracket notation is $\theta(\Omega_\omega, \Omega_2)$.

[1 /[1 $/_2$ 2] 2] has level $\theta(\Omega_\omega, \Omega_2)$,
[1 /[1 $/_2$ 3] 2] has level $\theta(\Omega_\omega, \Omega_2 2)$,
[1 /[1 $/_2$ 1 $/_2$ 2] 2] has level $\theta(\Omega_\omega, \Omega_2\hat{}2)$,
[1 /[1 [1 $/_2$ 2 $/_3$ 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_2\hat{}\Omega_2)$,
[1 /[1 [1 $/_3$ 3] 2] 2] has level $\theta(\Omega_\omega, \varepsilon(\Omega_2+1)) = \theta(\Omega_\omega, \theta_2(1))$,
[1 /[1 [1 [1 $/_3$ 2 $/_4$ 2] 2] 2] 2] has level $\theta(\Omega_\omega, \theta_2(\Omega_3))$,
[1 /[1 $\bullet_2$ 2] 2] has level $\theta(\Omega_\omega, \theta_2(\Omega_\omega))$,

[1 /[1 /[3 $_2$] 2] 2] has level $\theta(\Omega_\omega, \theta_2(\Omega_\omega, 1))$,

[1 /[1 /[1 $\bullet_2$ 2 $_2$] 2] 2] has level $\theta(\Omega_\omega, \theta_2(\Omega_\omega, \theta_2(\Omega_\omega)))$,

[1 /[1 /[1 /[1 $\bullet_2$ 2 $_2$] 2 $_2$] 2] 2] has level $\theta(\Omega_\omega, \theta_2(\Omega_\omega, \theta_2(\Omega_\omega, \theta_2(\Omega_\omega))))$.

The limit ordinal of the /[$_2$] bracket notation is $\theta(\Omega_\omega, \Omega_3)$.

[1 /[1 /[1 /$_3$ 2 $_2$] 2] 2] has level $\theta(\Omega_\omega, \Omega_3)$,

[1 /[1 $\bullet_2$ 2 /[1 /$_3$ 2 $_2$] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega))$,

[1 /[1 /[1 $\bullet_2$ 2 $_2$] 2 /[1 /$_3$ 2 $_2$] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \theta_2(\Omega_\omega)))$,

[1 /[1 /[1 /[1 $\bullet_2$ 2 $_2$] 2 $_2$] 2 /[1 /$_3$ 2 $_2$] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \theta_2(\Omega_\omega, \theta_2(\Omega_\omega))))$,

[1 /[1 /[1 /$_3$ 2 $_2$] 3] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3))$,

[1 /[1 /[1 /$_3$ 2 $_2$] 1 /[1 /$_3$ 2 $_2$] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3)\hat{}2)$,

[1 /[1 [2 /[1 /$_3$ 2 $_3$] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3)\hat{}\omega)$,

[1 /[1 [1 / 2 /[1 /$_3$ 2 $_3$] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3)\hat{}\Omega)$,

[1 /[1 [1 /$_2$ 2 /[1 /$_3$ 2 $_3$] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3)\hat{}\Omega_2)$,

[1 /[1 [1 $\bullet_2$ 2 /[1 /$_3$ 2 $_3$] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3)\hat{}\theta_2(\Omega_\omega))$,

[1 /[1 [1 /[1 $\bullet_2$ 2 $_2$] 2 /[1 /$_3$ 2 $_3$] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3)\hat{}\theta_2(\Omega_\omega, \theta_2(\Omega_\omega)))$,

[1 /[1 [1 /[1 /[1 $\bullet_2$ 2 $_2$] 2 $_2$] 2 /[1 /$_3$ 2 $_3$] 2] 2] 2] has level
$\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3)\hat{}\theta_2(\Omega_\omega, \theta_2(\Omega_\omega, \theta_2(\Omega_\omega))))$,

[1 /[1 [1 /[1 /$_3$ 2 $_2$] 2 /[1 /$_3$ 2 $_3$] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3)\hat{}\theta_2(\Omega_\omega, \Omega_3))$,

[1 /[1 [1 /[1 /$_3$ 2 $_2$] 1 /[1 /$_3$ 2 $_2$] 2 /[1 /$_3$ 2 $_3$] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3)\hat{}\hat{}3)$,

[1 /[1 [1 [1 /[1 /$_3$ 2 $_2$] 2 /[1 /$_3$ 2 $_3$] 2] 2 /[1 /$_3$ 2 $_3$] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3)\hat{}\hat{}4)$,

[1 /[1 [1 /$_3$ 2 /[1 /$_3$ 2 $_3$] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \varepsilon(\theta_2(\Omega_\omega, \Omega_3)+1))$,

[1 /[1 [1 [1 /$_3$ 2 /$_4$ 2] 2 /[1 /$_3$ 2 $_3$] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_3, \theta_2(\Omega_\omega, \Omega_3)+1))$,

[1 /[1 [1 [1 [1 /$_4$ 2 /$_5$ 2] 2] 2 /[1 /$_3$ 2 $_3$] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\theta_3(\Omega_4), \theta_2(\Omega_\omega, \Omega_3)+1))$,

[1 /[1 [1 $\bullet_3$ 2 /[1 /$_3$ 2 $_3$] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\theta_3(\Omega_\omega), \theta_2(\Omega_\omega, \Omega_3)+1))$,

[1 /[1 [1 /[1 $\bullet_2$ 2 $_3$] 2 /[1 /$_3$ 2 $_3$] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\theta_3(\Omega_\omega, \theta_2(\Omega_\omega)), \theta_2(\Omega_\omega, \Omega_3)+1))$,

[1 /[1 [1 /[1 /[1 $\bullet_2$ 2 $_2$] 2 $_3$] 2 /[1 /$_3$ 2 $_3$] 2] 2] 2] has level
$\theta(\Omega_\omega, \Omega_3 + \theta_2(\theta_3(\Omega_\omega, \theta_2(\Omega_\omega, \theta_2(\Omega_\omega))), \theta_2(\Omega_\omega, \Omega_3)+1))$,

[1 /[1 [1 /[1 /$_3$ 2 $_3$] 3] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\theta_3(\Omega_\omega, \Omega_3), 1))$
($\theta_2(\theta_3(\Omega_\omega, \Omega_3), 1)$ is the limit of $\theta_2(\alpha, \theta_2(\theta_3(\Omega_\omega, \Omega_3))+1)$ as $\alpha \to \theta_3(\Omega_\omega, \Omega_3)$),

[1 /[1 [1 /[1 /$_3$ 2 $_3$] 1 /$_3$ 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\theta_3(\Omega_\omega, \Omega_3)+1))$,

[1 /[1 [1 /[1 /$_3$ 2 $_3$] 1 /[1 /$_3$ 2 $_3$] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\theta_3(\Omega_\omega, \Omega_3)2))$,

[1 /[1 [1 [1 /[1 /$_3$ 2 $_3$] 2 /[1 /$_3$ 2 $_4$] 2] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\theta_3(\Omega_\omega, \Omega_3)\hat{}2))$,

[1 /[1 [1 [1 /$_4$ 2 /[1 /$_3$ 2 $_4$] 2] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\varepsilon(\theta_3(\Omega_\omega, \Omega_3)+1)))$,

[1 /[1 [1 [1 [1 /$_4$ 2 /$_5$ 2] 2 /[1 /$_3$ 2 $_4$] 2] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_4, \theta_3(\Omega_\omega, \Omega_3)+1))$,

[1 /[1 [1 [1 $\bullet_4$ 2 /[1 /$_3$ 2 $_4$] 2] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\theta_4(\Omega_\omega), \theta_3(\Omega_\omega, \Omega_3)+1))$,

[1 /[1 [1 [1 /[1 /$_3$ 2 $_4$] 3] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\theta_4(\Omega_\omega, \Omega_3), 1))$,

[1 /[1 [1 [1 [1 /[1 /$_3$ 2 $_5$] 3] 2] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\theta_5(\Omega_\omega, \Omega_3), 1))$,

[1 /[1 [1 [1 [1 [1 /[1 /$_3$ 2 $_6$] 3] 2] 2] 2] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\theta_6(\Omega_\omega, \Omega_3), 1))$,

[1 /[1 /[2 /$_3$ 2 $_2$] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3+1))$,

[1 /[1 /[3 /$_3$ 2 $_2$] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3+2))$,

[1 /[1 /[1 / 2 /$_3$ 2 $_2$] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3+\Omega))$,

[1 /[1 /[1 /$_2$ 2 /$_3$ 2 $_2$] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3+\Omega_2))$,

[1 /[1 /[1 $\bullet_2$ 2 /$_3$ 2 $_2$] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega)))$,

[1 /[1 /[1 /[1 /$_3$ 2 $_2$] 2 /$_3$ 2 $_2$] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3)))$,

[1 /[1 /[1 /[1 /[1 /$_3$ 2 $_2$] 2 /$_3$ 2 $_2$] 2 /$_3$ 2 $_2$] 2] 2] has level
$\theta(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3 + \theta_2(\Omega_\omega, \Omega_3))))$,

[1 /[1 /[1 /$_3$ 3 $_2$] 2] 2] has level $\theta(\Omega_\omega, \Omega_3 2)$,

[1 /[1 /[1 /$_3$ 1 /$_3$ 2 $_2$] 2] 2] has level $\theta(\Omega_\omega, \Omega_3\hat{}2)$,

$[1 /[1 /[1 [1 /_3 2 /_4 2] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_3\text{^}\Omega_3)$,
$[1 /[1 /[1 [1 /_4 3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \varepsilon(\Omega_3+1)) = \theta(\Omega_\omega, \theta_3(1))$,
$[1 /[1 /[1 [1 [1 /_4 2 /_5 2] 2] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \theta_3(\Omega_4))$,
$[1 /[1 /[1 \bullet_3 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \theta_3(\Omega_\omega))$,
$[1 /[1 /[1 /[3 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \theta_3(\Omega_\omega, 1))$,
$[1 /[1 /[1 /[1 \bullet_3 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \theta_3(\Omega_\omega, \theta_3(\Omega_\omega)))$,
$[1 /[1 /[1 /[1 /[1 \bullet_3 2 _3] 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \theta_3(\Omega_\omega, \theta_3(\Omega_\omega, \theta_3(\Omega_\omega))))$.

The limit ordinal of the $/[\ _3]$ bracket notation is $\theta(\Omega_\omega, \Omega_4)$.

$[1 /[1 /[1 /[1 /_4 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_4)$,
$[1 /[1 \bullet_2 2 /[1 /[1 /_4 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_4 + \theta_2(\Omega_\omega))$,
$[1 /[1 /[1 /[1 /_4 2 _3] 2 _2] 3] 2]$ has level $\theta(\Omega_\omega, \Omega_4 + \theta_2(\Omega_\omega, \Omega_4))$,
$[1 /[1 [1 /[1 /[1 /_4 2 _3] 2 _3] 3] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_4 + \theta_2(\theta_3(\Omega_\omega, \Omega_4), 1))$,
$[1 /[1 [1 [1 /[1 /[1 /_4 2 _3] 2 _4] 3] 2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_4 + \theta_2(\theta_4(\Omega_\omega, \Omega_4), 1))$,
$[1 /[1 /[2 /[1 /_4 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_4 + \theta_2(\Omega_\omega, \Omega_4+1))$,
$[1 /[1 /[1 /[1 /[1 /_4 2 _3] 2 _2] 2 /[1 /_4 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_4 + \theta_2(\Omega_\omega, \Omega_4 + \theta_2(\Omega_\omega, \Omega_4)))$,
$[1 /[1 /[1 /_3 2 /[1 /_4 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_4 + \Omega_3)$,
$[1 /[1 /[1 \bullet_3 2 /[1 /_4 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_4 + \theta_3(\Omega_\omega))$,
$[1 /[1 /[1 /[1 /_4 2 _3] 3 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_4 + \theta_3(\Omega_\omega, \Omega_4))$,
$[1 /[1 /[1 [1 /[1 /_4 2 _4] 3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_4 + \theta_3(\theta_4(\Omega_\omega, \Omega_4), 1))$,
$[1 /[1 /[1 [1 [1 /[1 /_4 2 _5] 3] 2] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_4 + \theta_3(\theta_5(\Omega_\omega, \Omega_4), 1))$,
$[1 /[1 /[1 /[2 /_4 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_4 + \theta_3(\Omega_\omega, \Omega_4+1))$,
$[1 /[1 /[1 /[1 /[1 /_4 2 _3] 2 /_4 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_4 + \theta_3(\Omega_\omega, \Omega_4 + \theta_3(\Omega_\omega, \Omega_4)))$,
$[1 /[1 /[1 /[1 /_4 3 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_4 2)$,
$[1 /[1 /[1 /[1 /_4 1 /_4 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_4\text{^}2)$,
$[1 /[1 /[1 /[1 [1 /_4 2 /_5 2] 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_4\text{^}\Omega_4)$,
$[1 /[1 /[1 /[1 [1 /_5 3] 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \varepsilon(\Omega_4+1)) = \theta(\Omega_\omega, \theta_4(1))$,
$[1 /[1 /[1 /[1 [1 [1 /_5 2 /_6 2] 2] 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \theta_4(\Omega_5))$,
$[1 /[1 /[1 /[1 \bullet_4 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \theta_4(\Omega_\omega))$,
$[1 /[1 /[1 /[1 /[3 _4] 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \theta_4(\Omega_\omega, 1))$,
$[1 /[1 /[1 /[1 /[1 \bullet_4 2 _4] 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \theta_4(\Omega_\omega, \theta_4(\Omega_\omega)))$,
$[1 /[1 /[1 /[1 /[1 /[1 \bullet_4 2 _4] 2 _4] 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \theta_4(\Omega_\omega, \theta_4(\Omega_\omega, \theta_4(\Omega_\omega))))$.

The limit ordinal of the $/[\ _4]$ bracket notation is $\theta(\Omega_\omega, \Omega_5)$.

A pattern can be seen:

$[1 /[1 /_2 2] 2]$ has level $\theta(\Omega_\omega, \Omega_2)$,
$[1 /[1 /[1 /_3 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_3)$,
$[1 /[1 /[1 /[1 /_4 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_4)$,
$[1 /[1 /[1 /[1 /[1 /_5 2 _4] 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_5)$,
$[1 /[1 /[1 /[1 /[1 /[1 /_6 2 _5] 2 _4] 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_6)$,
$[1 /[1 /[1 /[1 /[1 /[1 /[1 /_7 2 _6] 2 _5] 2 _4] 2 _3] 2 _2] 2] 2]$ has level $\theta(\Omega_\omega, \Omega_7)$.

The limit ordinal of the above is $\theta(\Omega_\omega+1)$ (limit of $\theta(\Omega_\omega, \Omega_n)$ as $n \to \omega$).

In order to proceed further, it is time to introduce another all-new special symbol – the double forward slash (//), which requires at least two pairs of square brackets enclosing it. The n-hyperseparator $/[X\ _n]$ symbol (for an array X and subscript n) is now rewritten as $[X // 2\ _n]$ – this means that $/_n$ is $[1 // 2\ _n]$ and $\bullet_n$ is $[2 // 2\ _n]$. This paves the way for n-hyperseparators of the form $[X_1 // X_2 // \ldots // X_{k\ n}]$ (for arbitrary strings X_i and $k \geq 2$) and beyond. When a string X contains at least one // symbol in its 'base layer', $[X\ _n] = [1\ [X\ _{n+1}]\ 2]$. The separator subscript is omitted whenever it is 1, as with slash subscripts.

The Angle Bracket Rules, as shown on pages 25-26 of Beyond Bird's Nested Arrays IV, now incorporate subscripts within the separator and angle bracket arrays. For example, in Rule A5, the separator [$A_{i,j}$] may be of the form [X_1 // X_2 // ... // X_k $_n$] for some i ≥ n, 1 ≤ j ≤ p_i and k ≥ 2. The remainder of array strings #* and $\#_i$ (for i ≥ 2) may include subscripts at the ends of them. Since the // symbol ranks higher than any n-hyperseparator or $/_n$ symbol for finite n (it may be regarded as an ω-hyperseparator), the #* string may begin with a // symbol as it is a '2- or higher order hyperseparator'.

An extra subrule within Angle Bracket Rule A5 is created as follows:-

Rule A5a* (separator [A_{i,p_i}] = [d $\#_H$ $_m$], where d ≥ 2 and $\#_H$ contains at least one // symbol in its 'base layer'):

S_i = 'b ‹$A_{i,1}$'› b [$A_{i,1}$] b ‹$A_{i,2}$'› b [$A_{i,2}$] ... b ‹A_{i,p_i-1}'› b [A_{i,p_i-1}] R_b [d $\#_H$ $_m$] c_i-1 $\#_i$',
R_n = 'b ‹R_{n-1}› b' (n > 1),
R_1 = 'b [d-1 $\#_H$ $_{m+b-1}$] b'.

Note that Rule A5a* with [A_{i,p_i}] = [2 // 2 $_m$] would mean that R_1 = 'b [1 // 2 $_{m+b-1}$] b' = 'b $/_{m+b-1}$ b'. Setting m = 1 gives R_1 = 'b $/_b$ b', which enables us to go beyond the $\theta(\Omega_\omega)$ ordinal.

Examples using Rule A5a* are:

{a, b [1 [5 // 2] 2] 2} = {a ‹0 [5 // 2] 2› b}
= {a ‹b ‹b ‹ ... ‹b ‹b [4 // 2 $_b$] b› b› ... › b› b› b}
(with b pairs of angle brackets),

{a, b [1 [1 [7 // 2 $_2$] 3] 2] 2} = {a ‹0 [1 [7 // 2 $_2$] 3] 2› b}
= {a ‹b ‹R_b [7 // 2 $_2$] 2› b› b}
(with R_n = 'b ‹R_{n-1}› b' and R_1 = 'b [6 // 2 $_{b+1}$] b')
= {a ‹b ‹b ‹b ‹ ... ‹b ‹b [6 // 2 $_{b+1}$] b› b› ... › b› b [7 // 2 $_2$] 2› b› b}
(with b+1 pairs of angle brackets).

The latter example required one application of Rule A5b (with m = 1, as [$A_{1,1}$] = [1 [7 // 2 $_2$] 3] is a 1-hyperseparator) prior to employing Rule A5a* (with [$A_{2,1}$] = [7 // 2 $_2$]).

Since the k ● symbols on the bottom of the first page of this document are shorthand for [2 // 2] symbols, each of the S strings making up the array {a ‹ S ● S ● ... ● S ● T ● c-1 # › b} are equal to

'b ‹1 // 2› b' = 'b ‹0 // 2› b [1 // 2] b ‹0 // 2› b [1 // 2] ... [1 // 2] b ‹0 // 2› b'
(with b 'b ‹0 // 2› b' strings)
= 'b / b / b / ... / b' (with b b's),

as the [1 // 2]'s become single forward slashes and 'b ‹0 // 2› b' = 'b', as 'b ‹0 #› b' = 'b' when # begins with a 2- or higher order hyperseparator. If the k ● symbols were each replaced by [d #*] separators (where d ≥ 2 and #* contains at least one // symbol in its 'base layer'), S = 'b ‹d-1 #*› b' and the $/_b$ symbol in the T string would be replaced by the [d-1 #* $_b$] separator.

If d in Rule A5a* (above) was equal to 1, in other words, the separator [A_{i,p_i}] = [1 $\#_H$ $_m$], where $\#_H$ contains at least one // symbol in its 'base layer', Rule A5b would apply, unless [A_{i,p_i}] = [1 // 2 $_m$] = $/_m$, in which case Rule A5a would apply. This is because [A_{i,p_i}] is an m-hyperseparator, where m ≥ 1. To take a simple example, when [X] is a normal separator,

{a, b [1 [1 [X] 2 // 2] 2] 2} = {a ‹0 [1 [X] 2 // 2] 2› b}
= {a ‹b ‹S_2› b› b} (Rule A5b, [1 [X] 2 // 2] is 1-hyperseparator)
= {a ‹b ‹b ‹X'› b // 2› b› b} (Rule A5c, [X] is 0-hyperseparator),

where X’ is identical to X apart from the first entry being reduced by 1. When [X] = [1] (comma),

$\{a, b\ \ [1\ [1, 2\ //\ 2]\ 2]\ \ 2\} = \{a \langle b \langle b\ //\ 2 \rangle\ b \rangle\ b\}$.

The $\theta(\Omega_\omega, \Omega_3)$ level separator

$\{a, b\ \ [1\ [1\ [1\ /_3\ 2\ //\ 2\ _2]\ 2\ //\ 2]\ 2]\ \ 2\} = \{a \langle 0\ [1\ [1\ /_3\ 2\ //\ 2\ _2]\ 2\ //\ 2]\ 2 \rangle\ b\}$

$= \{a \langle S_1 \rangle\ b\}$.

Since $p_1 = 1$, $c_1 = 2$, $\#_1 = \#^* =$ ‘’ (blank),

$[A_{1,1}] = [1\ [1\ /_3\ 2\ //\ 2\ _2]\ 2\ //\ 2]$ (1-hyperseparator),

$p_2 = 1$, $c_2 = 2$, $\#_2 =$ ‘// 2’,

$[A_{2,1}] = [1\ /_3\ 2\ //\ 2\ _2]$ (2-hyperseparator),

$p_3 = 1$, $c_3 = 2$, $\#_3 =$ ‘$//\ 2\ _2$’,

$[A_{3,1}] = /_3$ (3-hyperseparator),

by Rule A5b (m = 1),

$S_1 =$ ‘$b \langle S_2 \rangle\ b$’,

$t_1 = 1$, $t_2 = 0$, $t_3 = 0$;

by Rule A5b (m = 2),

$S_2 =$ ‘$b \langle S_3 \rangle\ b\ //\ 2$’,

$t_1 = 2$, $t_2 = 1$, $t_3 = 0$;

and by Rule A5a (m = 3, s = 3),

$S_3 =$ ‘$R_{b,3}$’,

$R_{n,3} =$ ‘$b \langle R_{n-1,3} \rangle\ b\ //\ 2\ _2$’,

$R_{1,3} =$ ‘0’.

It follows that,

$\{a, b\ \ [1\ [1\ [1\ /_3\ 2\ //\ 2\ _2]\ 2\ //\ 2]\ 2]\ \ 2\}$

$= \{a \langle b \langle b \langle b \langle b \langle \ldots \langle b \langle b\ //\ 2\ _2 \rangle\ b\ //\ 2\ _2 \rangle \ldots _2 \rangle\ b\ //\ 2\ _2 \rangle\ b\ //\ 2\ _2 \rangle\ b\ //\ 2 \rangle\ b \rangle\ b\}$

(with b+1 pairs of angle brackets and b //’s).

In general, for $n \ge 2$,

$[1\ [1\ [1\ [1\ [\ \ldots\ [1\ [1\ /_n\ 2\ //\ 2\ _{n-1}]\ 2\ //\ 2\ _{n-2}]\ \ldots\ _4]\ 2\ //\ 2\ _3]\ 2\ //\ 2\ _2]\ 2\ //\ 2]\ 2]$ has level $\theta(\Omega_\omega, \Omega_n)$.

As the ‘// 2’ series is exhausted at the $\theta(\Omega_\omega+1)$ level (limit of $\theta(\Omega_\omega, \Omega_n)$ as $n \to \omega$), we need to add one to the number after the double slash. This requires another new subrule within Angle Bracket Rule A5, in order to cater for the scenario when the separator $[A_{i,p_i}]$ can be written as a double slash.

The lowest separator containing ‘// 3’ is [1 [1 // 3] 2], which is at the $\theta(\Omega_\omega+1)$ level. It is defined as follows:

$\{a, b\ [1\ [1\ //\ 3]\ 2]\ 2\} = \{a \langle b \langle b \langle b \langle b \langle \ldots \langle b \langle b\ //\ 2\ _{b-1} \rangle\ b\ //\ 2\ _{b-2} \rangle \ldots _4 \rangle\ b\ //\ 2\ _3 \rangle\ b\ //\ 2\ _2 \rangle\ b\ //\ 2 \rangle\ b \rangle\ b\}$

(with b pairs of angle brackets).

As the new subrule is similar in many respects to Rule A5a (separator $[A_{i,p_i}] = /_m$, where $m \ge 1$), I refer to it as Rule A5a2. It is created as follows:-

Rule A5a2 (separator $[A_{i,p_i}] = //$):

$h =$ EndSub(‘$\#_i$’) (subscript at the end of $\#_i$, this is 1 by default),

$\#^*_i =$ DelEndSub(‘$\#_i$’) (identical to $\#_i$ but with the end subscript (h) deleted),

$S_i =$ ‘R_b’,

$R_n =$ ‘$b \langle A_{i,1}' \rangle\ b\ [A_{i,1}]\ b \langle A_{i,2}' \rangle\ b\ [A_{i,2}]\ \ldots\ b \langle A_{i,p_i-1}' \rangle\ b\ [A_{i,p_i-1}]\ b \langle R_{n-1} \rangle\ b\ \ //\ \ c_i\text{-}1\ \#^*_i\ _{h+b-n}$’ $(n > 1)$,

$R_1 =$ ‘0’.

The main difference between Rules A5a and A5a2 is that the R_n string ends in a subscript (h+b-n) under the latter subrule, where h is the subscript at the end of the $\#_i$ string, which is deleted in order to create the $\#^*_i$ string (h = 1 if the subscript is omitted from the end of $\#_i$). The h+b-n subscript is h for the R_b string, h+1 for the R_{b-1} string, h+2 for R_{b-2}, and so on, up to h+b-2 for the R_2 string (R_1 = ‘0’).

The $\theta(\Omega_\omega+1)$ level separator is verified using the new subrule:

{a, b [1 [1 // 3] 2] 2} = {a ‹0 [1 // 3] 2› b}
= {a ‹S_1› b}.

Since $p_1 = 1$, $c_1 = 2$, $\#_1 = \#^*$ = ‘’ (blank),
[$A_{1,1}$] = [1 // 3] (1-hyperseparator),
$p_2 = 1$, $c_2 = 3$, $\#_2$ = ‘’ (blank),
[$A_{2,1}$] = // (ω-hyperseparator),

by Rule A5b,

S_1 = ‘b ‹S_2› b’,

and by Rule A5a2 (h = 1, $\#^*_2$ = ‘’ (blank)),

S_2 = ‘R_b’,
R_n = ‘b ‹R_{n-1}› b // 2 $_{b+1-n}$’,
R_1 = ‘0’.

When a = b = 3, the above array becomes

{3, 3 [1 [1 // 3] 2] 2} = {3 ‹3 ‹3 ‹3 // 2 $_2$› 3 // 2› 3› 3}
= {3 ‹3 ‹ 3 ‹2 // 2 $_2$› 3 [3 // 2 $_2$] 3 ‹2 // 2 $_2$› 3 [3 // 2 $_2$] 3 ‹2 // 2 $_2$› 3 // 2 › 3› 3}
= {3 ‹3 ‹ 3 $/_2$ 3 $/_2$ 3 $\bullet_2$ 3 $/_2$ 3 $/_2$ 3 $\bullet_2$ 3 $/_2$ 3 $/_2$ 3 [3 // 2 $_2$]
3 $/_2$ 3 $/_2$ 3 $\bullet_2$ 3 $/_2$ 3 $/_2$ 3 $\bullet_2$ 3 $/_2$ 3 $/_2$ 3 [3 // 2 $_2$]
3 $/_2$ 3 $/_2$ 3 $\bullet_2$ 3 $/_2$ 3 $/_2$ 3 $\bullet_2$ 3 $/_2$ 3 $/_2$ 3 // 2 › 3› 3},

using Rules A2 and A6, with $\bullet_2$ as shorthand for [2 // 2 $_2$]. Under Rule A2,

‘3 ‹1 // 2 $_2$› 3’ = ‘3 $/_2$ 3 $/_2$ 3’

since the double slash (//) counts as a ‘2- or higher order hyperseparator’ and the [1 // 2 $_2$] separator ‘drops down’ to the $/_2$ symbol.

Separators from [1 [1 // 3] 2] to the next major milestone are as follows:

[1 [1 // 3] 2] has level $\theta(\Omega_\omega+1)$,
[1 [1 // 3] 3] has level $\theta(\theta_1(\Omega_\omega+1), 1)$,
[1 [1 // 3] 1 / 2] has level $\theta(\theta_1(\Omega_\omega+1)+1)$,
[1 [1 // 3] 1 [1 // 3] 2] has level $\theta(\theta_1(\Omega_\omega+1)2)$,
[1 [2 [1 // 3 $_2$] 2] 2] has level $\theta(\theta_1(\Omega_\omega+1)\omega)$,
[1 [1 / 2 [1 // 3 $_2$] 2] 2] has level $\theta(\theta_1(\Omega_\omega+1)\Omega)$,
[1 [1 [1 // 3] 2 [1 // 3 $_2$] 2] 2] has level $\theta(\theta_1(\Omega_\omega+1)$^2),
[1 [1 [1 // 3] 1 [1 // 3] 2 [1 // 3 $_2$] 2] 2] has level $\theta(\theta_1(\Omega_\omega+1)$^$\theta_1(\Omega_\omega+1))$,
[1 [1 [1 [1 // 3] 2 [1 // 3 $_2$] 2] 2 [1 // 3 $_2$] 2] 2] has level $\theta(\theta_1(\Omega_\omega+1)$^$\theta_1(\Omega_\omega+1)$^$\theta_1(\Omega_\omega+1))$,
[1 [1 $/_2$ 2 [1 // 3 $_2$] 2] 2] has level $\theta(\varepsilon(\theta_1(\Omega_\omega+1)+1))$,
[1 [1 $\bullet_2$ 2 [1 // 3 $_2$] 2] 2] has level $\theta(\theta_1(\theta_2(\Omega_\omega), \theta_1(\Omega_\omega+1)+1))$,
[1 [1 [1 $/_n$ 2 // 2 $_2$] 2 [1 // 3 $_2$] 2] 2] (1 ≤ n ≤ 3) has level $\theta(\theta_1(\theta_2(\Omega_\omega, \Omega_n), \theta_1(\Omega_\omega+1)+1))$,
[1 [1 [1 [1 $/_4$ 2 // 2 $_3$] 2 // 2 $_2$] 2 [1 // 3 $_2$] 2] 2] has level $\theta(\theta_1(\theta_2(\Omega_\omega, \Omega_4), \theta_1(\Omega_\omega+1)+1))$,
[1 [1 [1 // 3 $_2$] 3] 2] has level $\theta(\theta_2(\Omega_\omega+1), 1)$,
[1 [1 [1 // 3 $_2$] 1 $/_2$ 2] 2] has level $\theta(\theta_2(\Omega_\omega+1)+1)$,
[1 [1 [1 // 3 $_2$] 1 [1 // 3 $_2$] 2] 2] has level $\theta(\theta_2(\Omega_\omega+1)2)$,
[1 [1 [1 [1 // 3 $_2$] 2 [1 // 3 $_3$] 2] 2] 2] has level $\theta(\theta_2(\Omega_\omega+1)$^2),
[1 [1 [1 $/_3$ 2 [1 // 3 $_3$] 2] 2] 2] has level $\theta(\varepsilon(\theta_2(\Omega_\omega+1)+1))$,

[1 [1 [1 $\bullet_3$ 2 [1 // 3 $_3$] 2] 2] 2] has level $\theta(\theta_2(\theta_3(\Omega_\omega), \theta_2(\Omega_\omega+1)+1))$,
[1 [1 [1 [1 $/_n$ 2 // 2 $_3$] 2 [1 // 3 $_3$] 2] 2] 2] $(1 \le n \le 4)$ has level $\theta(\theta_2(\theta_3(\Omega_\omega, \Omega_n), \theta_2(\Omega_\omega+1)+1))$,
[1 [1 [1 [1 [1 $/_5$ 2 // 2 $_4$] 2 // 2 $_3$] 2 [1 // 3 $_3$] 2] 2] 2] has level $\theta(\theta_2(\theta_3(\Omega_\omega, \Omega_5), \theta_2(\Omega_\omega+1)+1))$,
[1 [1 [1 [1 // 3 $_3$] 3] 2] 2] has level $\theta(\theta_3(\Omega_\omega+1), 1)$,
[1 [1 [1 [1 [1 // 3 $_4$] 3] 2] 2] 2] has level $\theta(\theta_4(\Omega_\omega+1), 1)$,
[1 [1 [1 [1 [1 [1 // 3 $_5$] 3] 2] 2] 2] 2] has level $\theta(\theta_5(\Omega_\omega+1), 1)$,
[1 [2 // 3] 2] has level $\theta(\Omega_\omega+1, 1)$,
[1 [3 // 3] 2] has level $\theta(\Omega_\omega+1, 2)$,
[1 [1 / 2 // 3] 2] has level $\theta(\Omega_\omega+1, \Omega)$,
[1 [1 [1 // 3] 2 // 3] 2] has level $\theta(\Omega_\omega+1, \theta_1(\Omega_\omega+1))$,
[1 [1 $/_2$ 2 // 3] 2] has level $\theta(\Omega_\omega+1, \Omega_2)$,
[1 [1 [1 // 3 $_2$] 2 // 3] 2] has level $\theta(\Omega_\omega+1, \theta_2(\Omega_\omega+1))$,
[1 [1 [1 $/_3$ 2 // 3 $_2$] 2 // 3] 2] has level $\theta(\Omega_\omega+1, \Omega_3)$,
[1 [1 [1 [1 // 3 $_3$] 2 // 3 $_2$] 2 // 3] 2] has level $\theta(\Omega_\omega+1, \theta_3(\Omega_\omega+1))$,
[1 [1 [1 [1 $/_4$ 2 // 3 $_3$] 2 // 3 $_2$] 2 // 3] 2] has level $\theta(\Omega_\omega+1, \Omega_4)$,
[1 [1 [1 [1 [1 $/_5$ 2 // 3 $_4$] 2 // 3 $_3$] 2 // 3 $_2$] 2 // 3] 2] has level $\theta(\Omega_\omega+1, \Omega_5)$,
[1 [1 // 4] 2] has level $\theta(\Omega_\omega+2)$,
[1 [1 // 5] 2] has level $\theta(\Omega_\omega+3)$,
[1 [1 // 1 / 2] 2] has level $\theta(\Omega_\omega+\Omega)$,
[1 [1 // 1 $\bullet$ 2] 2] has level $\theta(\Omega_\omega+\theta_1(\Omega_\omega))$,
[1 [1 // 1 $/_2$ 2] 2] has level $\theta(\Omega_\omega+\Omega_2)$,
[1 [1 // 1 $\bullet_2$ 2] 2] has level $\theta(\Omega_\omega+\theta_2(\Omega_\omega))$,
[1 [1 // 1 [1 // 1 $/_3$ 2 $_2$] 2] 2] has level $\theta(\Omega_\omega+\Omega_3)$,
[1 [1 // 1 [1 // 1 $\bullet_3$ 2 $_2$] 2] 2] has level $\theta(\Omega_\omega+\theta_3(\Omega_\omega))$,
[1 [1 // 1 [1 // 1 [1 // 1 $/_4$ 2 $_3$] 2 $_2$] 2] 2] has level $\theta(\Omega_\omega+\Omega_4)$,
[1 [1 // 1 [1 // 1 [1 // 1 [1 // 1 $/_5$ 2 $_4$] 2 $_3$] 2 $_2$] 2] 2] has level $\theta(\Omega_\omega+\Omega_5)$.

[1 [1 // 1 // 2] 2] has level $\theta(\Omega_\omega 2)$,
[1 [1 // 1 // 3] 2] has level $\theta(\Omega_\omega 3)$,
[1 [1 // 1 // 1 / 2] 2] has level $\theta(\Omega_\omega \Omega)$,
[1 [1 // 1 // 1 $/_2$ 2] 2] has level $\theta(\Omega_\omega \Omega_2)$,
[1 [1 // 1 // 1 [1 // 1 // 1 $/_3$ 2 $_2$] 2] 2] has level $\theta(\Omega_\omega \Omega_3)$,
[1 [1 // 1 // 1 [1 // 1 // 1 [1 // 1 // 1 $/_4$ 2 $_3$] 2 $_2$] 2] 2] has level $\theta(\Omega_\omega \Omega_4)$,
[1 [1 // 1 // 1 // 2] 2] has level $\theta(\Omega_\omega\text{^}2)$,
[1 [1 // 1 // 1 // 1 // 2] 2] has level $\theta(\Omega_\omega\text{^}3)$,
[1 [1 [2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega\text{^}\omega)$ (// = [1 $//_2$ 2] just as / = [1 $/_2$ 2]),
[1 [2 [2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega\text{^}\omega, 1)$
(Rule A5a* also applies when the ‘base layer’ of $\#_H$ contains any ω-hyperseparator, but not when it contains a $//_2$ symbol as this is an (ω+1)-hyperseparator),
[1 [1 // 2 [2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega\text{^}\omega + 1)$,
[1 [1 // 1 // 2 [2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega\text{^}\omega + \Omega_\omega)$,
[1 [1 [2 $//_2$ 2] 3] 2] has level $\theta((\Omega_\omega\text{^}\omega)2)$,
[1 [1 [2 $//_2$ 2] 1 // 2] 2] has level $\theta(\Omega_\omega\text{^}(\omega+1))$,
[1 [1 [2 $//_2$ 2] 1 [2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega\text{^}(\omega 2))$,
[1 [1 [3 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega\text{^}\omega\text{^}2)$,
[1 [1 [1, 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega\text{^}\omega\text{^}\omega)$,
[1 [1 [1 [1 / 2] 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega\text{^}\varepsilon_0)$,
[1 [1 [1 [1 $\bullet$ 2] 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega\text{^}\theta(\Omega_\omega))$,
[1 [1 [1 [1 [1 [1 [1 $\bullet$ 2] 2 $//_2$ 2] 2] 2] 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega\text{^}\theta(\Omega_\omega\text{^}\theta(\Omega_\omega)))$,

[1 [1 [1 / 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^\Omega)$ (m in Rule A5b can now be ω),
[1 [1 [1 ● 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\theta_1(\Omega_\omega)})$,
[1 [1 [1 [1 [1 ● 2 $//_2$ 2] 2] 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\theta_1(\Omega_\omega^{\theta_1(\Omega_\omega)})})$,
[1 [1 [1 $/_2$ 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\Omega_2})$,
[1 [1 [1 $\bullet_2$ 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\theta_2(\Omega_\omega)})$,
[1 [1 [1 [1 [1 $\bullet_2$ 2 $//_2$ 2] 2 ${}_2$] 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\theta_2(\Omega_\omega^{\theta_2(\Omega_\omega)})})$,
[1 [1 [1 [1 [1 $/_3$ 2 $//_2$ 2] 2 ${}_2$] 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\Omega_3})$
(not [1 [1 [1 $/_3$ 2 $//_2$ 2] 2] 2] as we need to nest through [${}_2$] brackets),
[1 [1 [1 [1 [1 $\bullet_3$ 2 $//_2$ 2] 2 ${}_2$] 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\theta_3(\Omega_\omega)})$,
[1 [1 [1 [1 [1 [1 [1 $\bullet_3$ 2 $//_2$ 2] 2 ${}_3$] 2 $//_2$ 2] 2 ${}_2$] 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\theta_3(\Omega_\omega^{\theta_3(\Omega_\omega)})})$,
[1 [1 [1 [1 [1 [1 [1 $/_4$ 2 $//_2$ 2] 2 ${}_3$] 2 $//_2$ 2] 2 ${}_2$] 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\Omega_4})$,
[1 [1 [1 [1 [1 [1 [1 [1 [1 $/_5$ 2 $//_2$ 2] 2 ${}_4$] 2 $//_2$ 2] 2 ${}_3$] 2 $//_2$ 2] 2 ${}_2$] 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\Omega_5})$,
[1 [1 [1 [1 [1 [1 [1 [1 [1 [1 [1 $/_6$ 2 $//_2$ 2] 2 ${}_5$] 2 $//_2$ 2] 2 ${}_4$] 2 $//_2$ 2] 2 ${}_3$] 2 $//_2$ 2] 2 ${}_2$] 2 $//_2$ 2] 2] 2]
has level $\theta(\Omega_\omega^{\Omega_6})$.

[1 [1 [1 // 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\Omega_\omega})$,
[1 [1 [1 // 2 $//_2$ 2] 3] 2] has level $\theta((\Omega_\omega^{\Omega_\omega})2)$,
[1 [1 [1 // 2 $//_2$ 2] 1 [1 // 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{(\Omega_\omega 2)})$,
[1 [1 [2 // 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{(\Omega_\omega \omega)})$,
[1 [1 [1 / 2 // 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{(\Omega_\omega \Omega)})$,
[1 [1 [1 $/_2$ 2 // 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{(\Omega_\omega \Omega_2)})$,
[1 [1 [1 [1 [1 $/_3$ 2 // 2 $//_2$ 2] 2 ${}_2$] 2 // 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{(\Omega_\omega \Omega_3)})$,
[1 [1 [1 [1 [1 [1 [1 $/_4$ 2 // 2 $//_2$ 2] 2 ${}_3$] 2 // 2 $//_2$ 2] 2 ${}_2$] 2 // 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{(\Omega_\omega \Omega_4)})$,
[1 [1 [1 // 3 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\Omega_\omega^2})$,
[1 [1 [1 // 1 / 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\Omega_\omega^\Omega})$,
[1 [1 [1 // 1 $/_2$ 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\Omega_\omega^{\Omega_2}})$,
[1 [1 [1 // 1 [1 [1 // 1 $/_3$ 2 $//_2$ 2] 2 ${}_2$] 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\Omega_\omega^{\Omega_3}})$,
[1 [1 [1 // 1 [1 [1 // 1 [1 [1 // 1 $/_4$ 2 $//_2$ 2] 2 ${}_3$] 2 $//_2$ 2] 2 ${}_2$] 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\Omega_\omega^{\Omega_4}})$,
[1 [1 [1 // 1 // 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\Omega_\omega^{\Omega_\omega}})$,
[1 [1 [1 // 1 // 1 // 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\Omega_\omega^{\Omega_\omega^2}})$,
[1 [1 [1 [2 $//_2$ 2] 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\Omega_\omega^{\Omega_\omega^\omega}})$,
[1 [1 [1 [1 / 2 $//_2$ 2] 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\Omega_\omega^{\Omega_\omega^\Omega}})$,
[1 [1 [1 [1 // 2 $//_2$ 2] 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\Omega_\omega^{\Omega_\omega^{\Omega_\omega}}})$,
[1 [1 [1 [1 // 1 // 2 $//_2$ 2] 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\Omega_\omega^{\Omega_\omega^{\Omega_\omega^{\Omega_\omega}}}})$,
[1 [1 [1 [1 [1 // 2 $//_2$ 2] 2 $//_2$ 2] 2 $//_2$ 2] 2] 2] has level $\theta(\Omega_\omega^{\Omega_\omega^{\Omega_\omega^{\Omega_\omega^{\Omega_\omega^{\Omega_\omega}}}}})$.

The sequence of separators starting with the last three has limit ordinal $\theta(\varepsilon(\Omega_\omega+1)) = \theta(\theta_\omega(1))$. This ordinal is the proof theoretic ordinal of the subsystem Π^1_1-CA_0+BI of second-order arithmetic and is colloquially known as the Takeuti-Feferman-Buchholz ordinal. An example of a function whose growth rate is at the $\theta(\varepsilon(\Omega_\omega+1))$ level in the fast-growing hierarchy is Buchholz Hydras with ω labels. The growth rate of the Graph Minor Theorem (or Subcubic Graph Numbers) is upper bounded by this ordinal.

The $\theta(\theta_2(\Omega_\omega+1), 1)$ level separator
{a, b [1 [1 [1 // 3 ${}_2$] 3] 2] 2} = {a ‹0 [1 [1 // 3 ${}_2$] 3] 2› b}
= {a ‹S_1› b}.
Since $p_1 = 1$, $c_1 = 2$, $\#_1 = \#^* =$ ‘’ (blank),
[$A_{1,1}$] = [1 [1 // 3 ${}_2$] 3] (1-hyperseparator),
$p_2 = 1$, $c_2 = 3$, $\#_2 =$ ‘’ (blank),
[$A_{2,1}$] = [1 // 3 ${}_2$] (2-hyperseparator),

$p_3 = 1$, $c_3 = 3$, $\#_3$ = ‘$_2$’,

$[A_{3,1}] = //$ (ω-hyperseparator),

by Rule A5b (m = 1),

S_1 = ‘b ‹S_2› b’,

$t_1 = 1$, $t_2 = 0$;

by Rule A5b (m = 2),

S_2 = ‘b ‹S_3› b [1 // 3 $_2$] 2’,

$t_1 = 2$, $t_2 = 1$;

and by Rule A5a2 (h = 2, $\#^*_3$ = ‘’ (blank)),

S_3 = ‘R_b’,

R_n = ‘b ‹R_{n-1}› b // 2 $_{b+2-n}$’,

R_1 = ‘0’.

It follows that,

$$\{a, b\ \ [1\ [1\ [1\ //\ 3\ _2]\ 3]\ 2]\ \ 2\}$$

$$= \{a \langle b \langle b \langle b \langle b \langle \ldots \langle b \langle b\ //\ 2\ _b\rangle\ b\ //\ 2\ _{b-1}\rangle \ldots _4\rangle\ b\ //\ 2\ _3\rangle\ b\ //\ 2\ _2\rangle\ b\ [1\ //\ 3\ _2]\ 2\rangle\ b\rangle\ b\}$$

(with b+1 pairs of angle brackets).

The $\theta(\Omega_\omega\text{^}\Omega_3)$ level separator

$$\{a, b\ \ [1\ [1\ [1\ [1\ [1\ /_3\ 2\ //_2\ 2]\ 2\ _2]\ 2\ //_2\ 2]\ 2]\ 2]\ \ 2\} = \{a \langle 0\ [1\ [1\ [1\ [1\ /_3\ 2\ //_2\ 2]\ 2\ _2]\ 2\ //_2\ 2]\ 2]\ 2\rangle\ b\}$$

$$= \{a \langle S_1\rangle\ b\}.$$

Since $p_1 = 1$, $c_1 = 2$, $\#_1 = \#^*$ = ‘’ (blank),

$[A_{1,1}] = [1\ [1\ [1\ [1\ /_3\ 2\ //_2\ 2]\ 2\ _2]\ 2\ //_2\ 2]\ 2]$ (1-hyperseparator),

$p_2 = 1$, $c_2 = 2$, $\#_2$ = ‘’ (blank),

$[A_{2,1}] = [1\ [1\ [1\ /_3\ 2\ //_2\ 2]\ 2\ _2]\ 2\ //_2\ 2]$ (ω-hyperseparator),

$p_3 = 1$, $c_3 = 2$, $\#_3$ = ‘$//_2$ 2’,

$[A_{3,1}] = [1\ [1\ /_3\ 2\ //_2\ 2]\ 2\ _2]$ (2-hyperseparator),

$p_4 = 1$, $c_4 = 2$, $\#_4$ = ‘$_2$’,

$[A_{4,1}] = [1\ /_3\ 2\ //_2\ 2]$ (ω-hyperseparator),

$p_5 = 1$, $c_5 = 2$, $\#_5$ = ‘$//_2$ 2’,

$[A_{5,1}] = /_3$ (3-hyperseparator),

by Rule A5b (m = 1),

S_1 = ‘b ‹S_2› b’,

$t_1 = 1$, $t_2 = 0$, $t_3 = 0$, $t_\omega = 0$;

by Rule A5b (m = ω),

S_2 = ‘b ‹S_3› b’,

$t_1 = 2$, $t_2 = 1$, $t_3 = 1$, $t_\omega = 1$;

by Rule A5b (m = 2),

S_3 = ‘b ‹S_4› b $//_2$ 2’,

$t_1 = 3$, $t_2 = 2$, $t_3 = 0$, $t_\omega = 0$;

by Rule A5b (m = ω),

S_4 = ‘b ‹S_5› b $_2$’,

$t_1 = 4$, $t_2 = 3$, $t_3 = 1$, $t_\omega = 1$;

and by Rule A5a (m = 3, s = 4),

S_5 = ‘$R_{b,5}$’,

$R_{n,5}$ = ‘b ‹$R_{n-1,4}$› b $//_2$ 2’,

$R_{n,4}$ = ‘b ‹$R_{n,5}$› b $_2$’,

$R_{1,4}$ = ‘0’.

It follows that,

$$\{a, b\ \ [1\ [1\ [1\ [1\ [1\ /_3\ 2\ //_2\ 2]\ 2\ _2]\ 2\ //_2\ 2]\ 2]\ 2]\ \ 2\}$$

= {a ‹b ‹b ‹b ‹b ‹b ‹ ... ‹b ‹b ‹b $//_2$ 2› b $_2$› b $//_2$ 2› ... $_2$› b $//_2$ 2› b $_2$› b $//_2$ 2› b› b› b}
(with 2b+1 pairs of angle brackets and b $//_2$'s).

The $//_2$ symbol is an (ω+1)-hyperseparator. This means that separators containing $//_2$ in their 'base layers' (e.g. [X $//_2$ Y $_m$], where X and Y are arbitrary strings that may contain $/_n$, // or $//_2$ symbols) are ω-hyperseparators. The highest order hyperseparator contained within the 'base layer' of the $\#_H$ string in Rule A5a* is an ω-hyperseparator (the // symbol is just the lowest one). The [1 [1 [1 // 2 $//_2$ 2] 2] 2] separator (at $\theta(\Omega_\omega\text{^}\Omega_\omega)$ level) requires Rule A5a2 to be modified as follows:-

Rule A5a2 (separator $[A_{i,p_i}]$ = //):
s = i-t_ω,
h = EndSub('$\#_s$') (subscript at the end of $\#_s$, this is 1 by default),
$\#^*_s$ = DelEndSub('$\#_s$') (identical to $\#_s$ but with the end subscript (h) deleted),
$\#_{n,s}$ = '$\#^*_{s\ h+b-n}$' (1 < n < b),
$\#_{n,k}$ = '$\#_k$' (1 < n ≤ b, s < k ≤ i),
S_i = '$R_{b,i}$'.
For 1 < n ≤ b and s ≤ k < i,
$R_{n,i}$ = 'b ‹$A_{i,1}$'› b [$A_{i,1}$] b ‹$A_{i,2}$'› b [$A_{i,2}$] ... b ‹A_{i,p_i-1}'› b [A_{i,p_i-1}] b ‹$R_{n-1,s}$› b // c_i-1 $\#_{n,i}$',
$R_{n,k}$ = 'b ‹$A_{k,1}$'› b [$A_{k,1}$] b ‹$A_{k,2}$'› b [$A_{k,2}$] ... b ‹A_{k,p_k-1}'› b [A_{k,p_k-1}] b ‹$R_{n,k+1}$› b [A_{k,p_k}] c_k-1 $\#_{n,k}$',
$R_{1,s}$ = '0'.

The R_n string in the above subrule now becomes multilayered from $R_{n,s}$ to $R_{n,i}$, with the h+b-n subscript appearing at the end of the $R_{n,s}$ string.

The $\theta(\Omega_\omega\text{^}\Omega_\omega\text{^}\Omega_\omega\text{^}\Omega_\omega)$ level separator
{a, b [1 [1 [1 [1 // 2 $//_2$ 2] 2 $//_2$ 2] 2] 2] 2} = {a ‹0 [1 [1 [1 // 2 $//_2$ 2] 2 $//_2$ 2] 2] 2› b}
= {a ‹S_1› b}.
Since p_1 = 1, c_1 = 2, $\#_1$ = #* = '' (blank),
[$A_{1,1}$] = [1 [1 [1 // 2 $//_2$ 2] 2 $//_2$ 2] 2] (1-hyperseparator),
p_2 = 1, c_2 = 2, $\#_2$ = '' (blank),
[$A_{2,1}$] = [1 [1 // 2 $//_2$ 2] 2 $//_2$ 2] (ω-hyperseparator),
p_3 = 1, c_3 = 2, $\#_3$ = '$//_2$ 2',
[$A_{3,1}$] = [1 // 2 $//_2$ 2] (ω-hyperseparator),
p_4 = 1, c_4 = 2, $\#_4$ = '$//_2$ 2',
[$A_{4,1}$] = // (ω-hyperseparator),
by Rule A5b (m = 1),
S_1 = 'b ‹S_2› b',
t_1 = 1, t_ω = 0;
by Rule A5b (m = ω),
S_2 = 'b ‹S_3› b',
t_1 = 2, t_ω = 1;
by Rule A5b (m = ω),
S_3 = 'b ‹S_4› b $//_2$ 2',
t_1 = 3, t_ω = 2;
and by Rule A5a2 (s = 2, h = 1, $\#_{n,2}$ = '$_{b+1-n}$'),
S_4 = '$R_{b,4}$',
$R_{n,4}$ = 'b ‹$R_{n-1,2}$› b $//_2$ 2',
$R_{n,2}$ = 'b ‹$R_{n,3}$› b $_{b+1-n}$',
$R_{n,3}$ = 'b ‹$R_{n,4}$› b $//_2$ 2' (so, S_2 = $R_{b,2}$),

$R_{1,2}$ = ‘0’.

It follows that,

{a, b [1 [1 [1 [1 // 2 $//_2$ 2] 2 $//_2$ 2] 2] 2] 2}

= {a ‹b ‹R_b› b› b} (with R_n = ‘b ‹b ‹b ‹R_{n-1}› b $//_2$ 2› b $//_2$ 2› b $_{b+1-n}$’ and R_1 = ‘0’)

= {a ‹b ‹b ‹b ‹b ‹b ‹b ‹b ‹ ... ‹b ‹b ‹b ‹b ‹b $//_2$ 2› b $//_2$ 2› b $_{b-1}$› b $//_2$ 2› b $//_2$ 2› ...
$_3$› b $//_2$ 2› b $//_2$ 2› b $_2$› b $//_2$ 2› b $//_2$ 2› b› b› b}

(with 3b-2 pairs of angle brackets and 2b-2 $//_2$’s).

The $\theta(\Omega_\omega\text{^}\Omega_\omega)$ level separator

{a, b [1 [1 [1 // 2 $//_2$ 2] 2] 2] 2} = {a ‹0 [1 [1 // 2 $//_2$ 2] 2] 2› b}

= {a ‹S_1› b}

is similar to the $\theta(\Omega_\omega\text{^}\Omega_\omega\text{^}\Omega_\omega\text{^}\Omega_\omega)$ level separator (above) except that [$A_{i,1}$] for the lower separator is [$A_{i+1,1}$] for the higher one, Rule A5b is executed twice (once with m = 1 and once with m = ω, leaving t_1 = 2 and t_ω = 1) prior to finishing with Rule A5a2 (with s = 2, h = 1, $\#_{n,2}$ = ‘$_{b+1-n}$’), which results in

S_3 = ‘$R_{b,3}$’,

$R_{n,3}$ = ‘b ‹$R_{n-1,2}$› b $//_2$ 2’,

$R_{n,2}$ = ‘b ‹$R_{n,3}$› b $_{b+1-n}$’,

$R_{1,2}$ = ‘0’.

It follows that,

{a, b [1 [1 [1 // 2 $//_2$ 2] 2] 2] 2}

= {a ‹b ‹b ‹b ‹b ‹b ‹ ... ‹b ‹b ‹b $//_2$ 2› b $_{b-1}$› b $//_2$ 2› ... $_3$› b $//_2$ 2› b $_2$› b $//_2$ 2› b› b› b}

(with 2b-1 pairs of angle brackets and b-1 $//_2$’s).

Further modifications to the above subrule will be needed in order to cater for [$A_{i,pi}$] being the $//_2$ symbol, and for higher order $//_n$ symbols, which are introduced in the next stage of the development of our separators.

[1 [1 [1 $//_2$ 3] 2] 2] has level $\theta(\varepsilon(\Omega_\omega+1)) = \theta(\theta_\omega(1))$,
[1 [1 [1 $//_2$ 3] 3] 2] has level $\theta(\varepsilon(\Omega_\omega+1), 1)$,
[1 [1 [1 $//_2$ 3] 1 / 2] 2] has level $\theta(\varepsilon(\Omega_\omega+1), \Omega)$,
[1 [1 [1 $//_2$ 3] 1 // 2] 2] has level $\theta(\varepsilon(\Omega_\omega+1)+1)$,
[1 [1 [1 $//_2$ 3] 1 [1 $//_2$ 3] 2] 2] has level $\theta(\varepsilon(\Omega_\omega+1)2)$,
[1 [1 [2 $//_2$ 3] 2] 2] has level $\theta(\varepsilon(\Omega_\omega+1)\omega)$,
[1 [1 [1 / 2 $//_2$ 3] 2] 2] has level $\theta(\varepsilon(\Omega_\omega+1)\Omega)$,
[1 [1 [1 // 2 $//_2$ 3] 2] 2] has level $\theta(\varepsilon(\Omega_\omega+1)\Omega_\omega)$,
[1 [1 [1 [1 $//_2$ 3] 2 $//_2$ 3] 2] 2] has level $\theta(\varepsilon(\Omega_\omega+1)\text{^}2)$,
[1 [1 [1 [1 $//_2$ 3] 1 [1 $//_2$ 3] 2 $//_2$ 3] 2] 2] has level $\theta(\varepsilon(\Omega_\omega+1)\text{^}\varepsilon(\Omega_\omega+1))$,
[1 [1 [1 [1 [1 $//_2$ 3] 2 $//_2$ 3] 2 $//_2$ 3] 2] 2] has level $\theta(\varepsilon(\Omega_\omega+1)\text{^}\varepsilon(\Omega_\omega+1)\text{^}\varepsilon(\Omega_\omega+1))$,
[1 [1 [1 $//_2$ 4] 2] 2] has level $\theta(\varepsilon(\Omega_\omega+2)) = \theta(\theta_\omega(1, 1))$,
[1 [1 [1 $//_2$ 5] 2] 2] has level $\theta(\varepsilon(\Omega_\omega+3)) = \theta(\theta_\omega(1, 2))$,
[1 [1 [1 $//_2$ 1 / 2] 2] 2] has level $\theta(\varepsilon(\Omega_\omega+\Omega)) = \theta(\theta_\omega(1, \Omega))$,
[1 [1 [1 $//_2$ 1 // 2] 2] 2] has level $\theta(\varepsilon(\Omega_\omega 2)) = \theta(\theta_\omega(1, \Omega_\omega))$,
[1 [1 [1 $//_2$ 1 // 1 // 2] 2] 2] has level $\theta(\varepsilon(\Omega_\omega\text{^}2))$,
[1 [1 [1 $//_2$ 1 [1 // 2 $//_2$ 2] 2] 2] 2] has level $\theta(\varepsilon(\Omega_\omega\text{^}\Omega_\omega))$,
[1 [1 [1 $//_2$ 1 [1 $//_2$ 3] 2] 2] 2] has level $\theta(\varepsilon(\varepsilon(\Omega_\omega+1)))$,
[1 [1 [1 $//_2$ 1 [1 $//_2$ 1 [1 $//_2$ 3] 2] 2] 2] 2] has level $\theta(\varepsilon(\varepsilon(\varepsilon(\Omega_\omega+1))))$,
[1 [1 [1 $//_2$ 1 $//_2$ 2] 2] 2] has level $\theta(\zeta(\Omega_\omega+1)) = \theta(\theta_\omega(2))$,
[1 [1 [1 $//_2$ 1 $//_2$ 1 $//_2$ 2] 2] 2] has level $\theta(\theta_\omega(3))$,
[1 [1 [1 [2 $//_3$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\omega))$ (in general, $//_n$ = [1 $//_{n+1}$ 2]),

[1 [1 [1 [1 / 2 $//_3$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\Omega))$,
[1 [1 [1 [1 // 2 $//_3$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_\omega))$,
[1 [1 [1 [1 [1 $//_2$ 3] 2 $//_3$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\theta_\omega(1)))$,
[1 [1 [1 [1 [1 [1 [1 $//_2$ 3] 2 $//_3$ 2] 2] 2 $//_3$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\theta_\omega(\theta_\omega(1))))$.

[1 [1 [1 [1 $//_2$ 2 $//_3$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega+1}) = \theta(\theta_\omega(\Omega_{\omega+1}))$,
[1 [1 [1 [1 $//_2$ 2 $//_3$ 2] 2] 3] 2] has level $\theta(\theta_\omega(\Omega_{\omega+1}), 1)$,
[1 [1 [1 [1 $//_2$ 2 $//_3$ 2] 2] 1 / 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega+1}), \Omega)$,
[1 [1 [1 [1 $//_2$ 2 $//_3$ 2] 2] 1 // 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega+1})+1)$,
[1 [1 [2 [1 $//_2$ 2 $//_3$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega+1})\omega)$,
[1 [1 [1 / 2 [1 $//_2$ 2 $//_3$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega+1})\Omega)$,
[1 [1 [1 // 2 [1 $//_2$ 2 $//_3$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega+1})\Omega_\omega)$,
[1 [1 [1 $//_2$ 2 [1 $//_2$ 2 $//_3$ 2] 2] 2] 2] has level $\theta(\varepsilon(\theta_\omega(\Omega_{\omega+1})+1))$,
[1 [1 [1 [1 $//_2$ 2 $//_3$ 2] 3] 2] 2] has level $\theta(\Omega_{\omega+1}, 1) = \theta(\theta_\omega(\Omega_{\omega+1}, 1))$
($\theta_\omega(\Omega_{\omega+1}, 1)$ is the limit of $\theta_\omega(\alpha, \theta_\omega(\Omega_{\omega+1})+1)$ as $\alpha \to \Omega_{\omega+1}$),
[1 [1 [1 [1 $//_2$ 2 $//_3$ 2] 1 / 2] 2] 2] has level $\theta(\Omega_{\omega+1}, \Omega)$,
[1 [1 [1 [1 $//_2$ 2 $//_3$ 2] 1 // 2] 2] 2] has level $\theta(\Omega_{\omega+1}, \Omega_\omega)$,
[1 [1 [1 [1 $//_2$ 2 $//_3$ 2] 1 $//_2$ 2] 2] 2] has level $\theta(\Omega_{\omega+1}+1)$,
[1 [1 [1 [2 $//_2$ 2 $//_3$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega+1}\omega)$,
[1 [1 [1 [1 / 2 $//_2$ 2 $//_3$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega+1}\Omega)$,
[1 [1 [1 [1 // 2 $//_2$ 2 $//_3$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega+1}\Omega_\omega)$,
[1 [1 [1 [1 $//_2$ 3 $//_3$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega+1}$^$2)$,
[1 [1 [1 [1 $//_2$ 1 $//_2$ 2 $//_3$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega+1}$^$\Omega_{\omega+1})$,
[1 [1 [1 [1 [1 $//_2$ 2 $//_3$ 2] 2 $//_3$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega+1}$^$\Omega_{\omega+1}$^$\Omega_{\omega+1})$.

[1 [1 [1 [1 $//_3$ 3] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega+1}+1)) = \theta(\theta_{\omega+1}(1))$,
[1 [1 [1 [1 // 2 $//_3$ 3] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega+1}+1)\Omega_\omega)$,
[1 [1 [1 [1 $//_2$ 2 $//_3$ 3] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega+1}+1)\Omega_{\omega+1})$,
[1 [1 [1 [1 [1 $//_3$ 3] 2 $//_3$ 3] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega+1}+1)$^$2)$,
[1 [1 [1 [1 $//_3$ 4] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega+1}+2)) = \theta(\theta_{\omega+1}(1, 1))$,
[1 [1 [1 [1 $//_3$ 1 // 2] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega+1}+\Omega_\omega)) = \theta(\theta_{\omega+1}(1, \Omega_\omega))$,
[1 [1 [1 [1 $//_3$ 1 $//_2$ 2] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega+1}2)) = \theta(\theta_{\omega+1}(1, \Omega_{\omega+1}))$,
[1 [1 [1 [1 $//_3$ 1 [1 $//_3$ 3] 2] 2] 2] 2] has level $\theta(\varepsilon(\varepsilon(\Omega_{\omega+1}+1)))$,
[1 [1 [1 [1 $//_3$ 1 $//_3$ 2] 2] 2] 2] has level $\theta(\zeta(\Omega_{\omega+1}+1)) = \theta(\theta_{\omega+1}(2))$,
[1 [1 [1 [1 [2 $//_4$ 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega+1}(\omega))$,
[1 [1 [1 [1 [1 // 2 $//_4$ 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega+1}(\Omega_\omega))$,
[1 [1 [1 [1 [1 $//_2$ 2 $//_4$ 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega+1}(\Omega_{\omega+1}))$,
[1 [1 [1 [1 [1 [1 $//_3$ 3] 2 $//_4$ 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega+1}(\theta_{\omega+1}(1)))$,
[1 [1 [1 [1 [1 [1 [1 [1 $//_3$ 3] 2 $//_4$ 2] 2] 2 $//_4$ 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega+1}(\theta_{\omega+1}(\theta_{\omega+1}(1))))$.

Continuing this pattern, I would find that:
[1 [1 [1 [1 [1 $//_3$ 2 $//_4$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega+2}) = \theta(\theta_{\omega+1}(\Omega_{\omega+2}))$,
[1 [1 [1 [1 [1 $//_4$ 3] 2] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega+2}+1)) = \theta(\theta_{\omega+2}(1))$,
[1 [1 [1 [1 [1 $//_4$ 1 $//_4$ 2] 2] 2] 2] 2] has level $\theta(\zeta(\Omega_{\omega+2}+1)) = \theta(\theta_{\omega+2}(2))$,
[1 [1 [1 [1 [1 [2 $//_5$ 2] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega+2}(\omega))$,
[1 [1 [1 [1 [1 [1 $//_4$ 2 $//_5$ 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega+3})$,
[1 [1 [1 [1 [1 [1 $//_5$ 3] 2] 2] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega+3}+1)) = \theta(\theta_{\omega+3}(1))$,
[1 [1 [1 [1 [1 [1 $//_5$ 1 $//_5$ 2] 2] 2] 2] 2] 2] has level $\theta(\zeta(\Omega_{\omega+3}+1)) = \theta(\theta_{\omega+3}(2))$,
[1 [1 [1 [1 [1 [1 [2 $//_6$ 2] 2] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega+3}(\omega))$,

[1 [1 [1 [1 [1 [1 [1 $//_5$ 2 $//_6$ 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega+4})$,
[1 [1 [1 [1 [1 [1 [1 [1 $//_6$ 2 $//_7$ 2] 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega+5})$,
[1 [1 [1 [1 [1 [1 [1 [1 [1 $//_7$ 2 $//_8$ 2] 2] 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega+6})$.

The sequence of separators starting with the last three has limit ordinal $\theta(\Omega_{\omega2})$. This is similar to the pattern on page 23 of Beyond Bird's Nested Arrays IV with single forward slashes except that each separator in the above pattern sports an extra pair of square brackets and the associated ordinal levels have ω-1 added to the values in the Ω and θ function subscripts.

The generalised $//_n$ symbol is an (ω+n-1)-hyperseparator, with a minimum of n+1 pairs of square brackets in a curly bracket array. The initial part of Rule A5 is altered to take account of double slash symbols. Take x_1 to be the highest subscript to a single slash or closed square bracket within $[A_{1,p_1}]$. If there are any double slashes within $[A_{1,p_1}]$, set $x = \omega+x_2-1$, where x_2 is the highest subscript to a double slash within $[A_{1,p_1}]$, otherwise set $x = x_1$ (and regard $x_2 = 0$). The number of tally counters (t-counters) required is x_1+x_2 (x_1 below the ωth t-counter and x_2 from the ωth t-counter onwards). The maximum value of this, or of x, has now been doubled from ω to ω2. The t_α counter ($1 \le \alpha < \omega2$) tallies the number of successive applications of Rule A5b where $[A_{i,p_i}]$ is an α- or higher order hyperseparator and is reset to 0 whenever $[A_{i,p_i}]$ is a lower order hyperseparator.

The separator $[A_{i,p_i}]$ can now be $//_m$, for any finite m ≥ 1, for some i ≥ m+1 and $p_i \ge 1$. The subscript at the end of the $R_{n,s}$ string (end of the $\#_{n,s}$ string) is h+b-n when m = 1 and h for higher values of m. Rule A5a2 is modified as follows:-

Rule A5a2 (separator $[A_{i,p_i}] = //_m$, where m ≥ 1):

$s = i-t_{\omega+m-1}$,
h = EndSub('$\#_s$') (subscript at the end of $\#_s$, this is 1 by default),
$\#^*_s$ = DelEndSub('$\#_s$') (identical to $\#_s$ but with the end subscript (h) deleted),
$\#_{n,s}$ = '$\#^*_{s\ h+b-n}$' (1 < n < b, m = 1),
$\#_{n,k}$ = '$\#_k$' (1 < n ≤ b and either s < k ≤ i or k = s, m ≥ 2),
S_i = '$R_{b,i}$'.

For 1 < n ≤ b and s ≤ k < i,

$R_{n,i}$ = 'b ‹$A_{i,1}$'› b [$A_{i,1}$] b ‹$A_{i,2}$'› b [$A_{i,2}$] ... b ‹A_{i,p_i-1}'› b [A_{i,p_i-1}] b ‹$R_{n-1,s}$› b $//_m$ c_i-1 $\#_{n,i}$',
$R_{n,k}$ = 'b ‹$A_{k,1}$'› b [$A_{k,1}$] b ‹$A_{k,2}$'› b [$A_{k,2}$] ... b ‹A_{k,p_k-1}'› b [A_{k,p_k-1}] b ‹$R_{n,k+1}$› b [A_{k,p_k}] c_k-1 $\#_{n,k}$',
$R_{1,s}$ = '0'.

Take $[X_k]$ = [1 [1 [... [1 [1 $//_k$ 3] 2] ...] 2] 2] (with k pairs of square brackets).

The $\theta(\varepsilon(\Omega_{\omega+k-2}+1))$ level separator (k ≥ 2)

{a, b [1 [X_k] 2] 2} = {a ‹0 [X_k] 2› b}
= {a ‹S_1› b}.

Since $p_1 = 1$, $c_1 = 2$, $\#_1 = \#^*$ = '' (blank),
[$A_{1,1}$] = [X_k] (1-hyperseparator),
$p_j = 1$, $c_j = 2$, $\#_j$ = '' (blank),
[$A_{j,1}$] = [1 [1 [... [1 [1 $//_k$ 3] 2] ...] 2] 2]
((ω+j-2)-hyperseparator, with k-j+1 pairs of square brackets),

for 2 ≤ j ≤ k, and

$p_{k+1} = 1$, $c_{k+1} = 3$, $\#_{k+1}$ = '' (blank),
[$A_{k+1,1}$] = $//_k$ ((ω+k-1)-hyperseparator),

by the jth of k applications of Rule A5b (m = 1 when j = 1 and m = ω+j-2 when 2 ≤ j ≤ k),

S_j = 'b ‹S_{j+1}› b',
$t_1 = j$, $t_\omega = j-1$, $t_{\omega+1} = j-2$, $t_{\omega+2} = j-3$, ... , $t_{\omega+j-2} = 1$, $t_{\omega+j-1} = t_{\omega+j} = ... = t_{\omega+k-1} = 0$

(t-counters finish on t_1 = k, t_ω = k-1, $t_{\omega+1}$ = k-2, $t_{\omega+2}$ = k-3, ... , $t_{\omega+k-2}$ = 1, $t_{\omega+k-1}$ = 0),
and by Rule A5a2 (m = k, s = k+1; the $R_{n,k}$ string is disregarded as s = i; $\#_{n,s}$ = '$\#_s$' as m ≥ 2),

S_{k+1} = '$R_{b,k+1}$',
$R_{n,k+1}$ = 'b ‹$R_{n-1,k+1}$› b $//_k$ 2',
$R_{1,k+1}$ = '0'.

It follows that,

{a, b [1 [X_k] 2] 2} = {a ‹b ‹b ‹ ... ‹b ‹S› b› ... › b› b› b}
(with k pairs of angle brackets),

where S = 'b ‹b ‹b ‹ ... ‹b ‹b $//_k$ 2› b $//_k$ 2› ... › b $//_k$ 2› b $//_k$ 2› b'
(with b-1 pairs of angle brackets and $//_k$'s).

When a = 3, b = 2 and k = 2, the above array (with the θ(ε(Ω_ω+1)) level separator) becomes

{3, 2 [1 [1 [1 $//_2$ 3] 2] 2] 2}
= {3 ‹2 ‹2 ‹2 $//_2$ 2› 2› 2› 2}
= {3 ‹2 ‹ 2 ‹1 $//_2$ 2› 2 [2 $//_2$ 2] 2 ‹1 $//_2$ 2› 2 › 2› 2}
= {3 ‹2 ‹ 2 // 2 [2 $//_2$ 2] 2 // 2 › 2› 2}
= {3 ‹ 2 ‹ 1 // 2 [2 $//_2$ 2] 2 // 2 › 2 [2 // 2 [2 $//_2$ 2] 2 // 2] 2 ‹ 1 // 2 [2 $//_2$ 2] 2 // 2 › 2 › 2}
= {3 ‹ 2 [1 // 2 [2 $//_2$ 2] 2 // 2] 2 [2 // 2 [2 $//_2$ 2] 2 // 2] 2 [1 // 2 [2 $//_2$ 2] 2 // 2] 2 › 2},

using Rules A2 and A6. The [1 $//_2$ 2] symbol 'drops down' to a double slash (//).

The $//_n$ symbol may either be regarded as an (ω+n-1)-hyperseparator or an (n, 2)-hyperseparator, which is an n-hyperseparator on the second level in the extension to my Hierarchical Hyper-Nested Array Notation. This is because ω+n-1 is the ordinal level of the [n, 2] separator. Thus, the // symbol is a (1, 2)-hyperseparator if one wished to avoid using transfinite numbers in describing hyperlevels (hyperseparator levels); I think that it is better to use an array system with finite integers when referring to these levels as I extend my notation. The highest order hyperseparator contained within the 'base layer' of the $\#_H$ string in Rule A5a* has hyperlevel (1, 2); a (1, 2)-hyperseparator is either a double slash or a separator with at least one $//_n$ symbol enclosed by n-1 pairs of square brackets for some n ≥ 2, but with no $//_n$ symbols enclosed by fewer than n-1 pairs of square brackets for any n. A separator array that is not itself an (n, 2)-hyperseparator (for any n) but has, in its 'base layer', one or more (1, 2)-hyperseparators may carry a subscript, like slash symbols ([1 // 2 $_n$] 'drops down' to $/_n$). An (n, 1)-hyperseparator is an n-hyperseparator (remove trailing 1's as with arrays).

Now I will bring in another all-new 1-hyperseparator symbol – the double black circle (●●). The smallest separator containing this symbol is the [1 ●● 2] separator.

The θ($\Omega_{\omega2}$) level separator

{a, b [1 ●● 2] 2} = {a ‹0 ●● 2› b}
= {a ‹b ‹b ‹ ... ‹b ‹b $//_b$ b› b› ... › b› b› b},
(with b+1 pairs of angle brackets).

This is equivalent to

{a, b [1 [1 [... [1 [1 $//_b$ 1, 2] 2] ...] 2] 2] 2} (with b+1 pairs of square brackets).

[1 ●● 2] has level θ($\Omega_{\omega2}$),
[1 [1 ●● 2] 2 ●● 2] has level θ($\Omega_{\omega2}$)2,
[1 / 2 ●● 2] has level ε(θ($\Omega_{\omega2}$)+1) = θ(1, θ($\Omega_{\omega2}$)+1),
[1 ● 2 ●● 2] has level θ(θ_1(Ω_ω), θ($\Omega_{\omega2}$)+1),
[1 [1 // 3] 2 ●● 2] has level θ(θ_1(Ω_ω+1), θ($\Omega_{\omega2}$)+1),
[1 [1 [1 $//_2$ 3] 2] 2 ●● 2] has level θ(θ_1(ε(Ω_ω+1)), θ($\Omega_{\omega2}$)+1),

[1 [1 [1 [1 $//_2$ 2 $//_3$ 2] 2] 2] 2 ●● 2] has level $\theta(\theta_1(\Omega_{\omega+1}), \theta(\Omega_{\omega2})+1)$,
[1 [1 [1 [1 [1 $//_3$ 2 $//_4$ 2] 2] 2] 2] 2 ●● 2] has level $\theta(\theta_1(\Omega_{\omega+2}), \theta(\Omega_{\omega2})+1)$,
[1 ●● 3] has level $\theta(\theta_1(\Omega_{\omega2}), 1)$,
[1 ●● 1 / 2] has level $\theta(\theta_1(\Omega_{\omega2})+1)$,
[1 ●● 1 ● 2] has level $\theta(\theta_1(\Omega_{\omega2})+\theta_1(\Omega_{\omega}))$,
[1 ●● 1 ●● 2] has level $\theta(\theta_1(\Omega_{\omega2})2)$,
[1 [2 $\bullet\bullet_2$ 2] 2] has level $\theta(\theta_1(\Omega_{\omega2})\omega)$,
[1 [1 / 2 $\bullet\bullet_2$ 2] 2] has level $\theta(\theta_1(\Omega_{\omega2})\Omega)$,
[1 [1 ● 2 $\bullet\bullet_2$ 2] 2] has level $\theta(\theta_1(\Omega_{\omega2})\theta_1(\Omega_{\omega}))$,
[1 [1 ●● 2 $\bullet\bullet_2$ 2] 2] has level $\theta(\theta_1(\Omega_{\omega2})\text{^}2)$,
[1 [1 $/_2$ 2 $\bullet\bullet_2$ 2] 2] has level $\theta(\varepsilon(\theta_1(\Omega_{\omega2})+1))$,
[1 [1 $\bullet_2$ 2 $\bullet\bullet_2$ 2] 2] has level $\theta(\theta_1(\theta_2(\Omega_{\omega}), \theta_1(\Omega_{\omega2})+1))$,
[1 [1 [1 // 3 $_2$] 2 $\bullet\bullet_2$ 2] 2] has level $\theta(\theta_1(\theta_2(\Omega_{\omega}+1), \theta_1(\Omega_{\omega2})+1))$,
[1 [1 [1 [1 $//_2$ 3] 2 $_2$] 2 $\bullet\bullet_2$ 2] 2] has level $\theta(\theta_1(\theta_2(\varepsilon(\Omega_{\omega}+1)), \theta_1(\Omega_{\omega2})+1))$,
[1 [1 [1 [1 [1 $//_2$ 2 $//_3$ 2] 2] 2 $_2$] 2 $\bullet\bullet_2$ 2] 2] has level $\theta(\theta_1(\theta_2(\Omega_{\omega+1}), \theta_1(\Omega_{\omega2})+1))$,
[1 [1 [1 [1 [1 [1 $//_3$ 2 $//_4$ 2] 2] 2] 2 $_2$] 2 $\bullet\bullet_2$ 2] 2] has level $\theta(\theta_1(\theta_2(\Omega_{\omega+2}), \theta_1(\Omega_{\omega2})+1))$,
[1 [1 $\bullet\bullet_2$ 3] 2] has level $\theta(\theta_2(\Omega_{\omega2}), 1)$,
[1 [1 $\bullet\bullet_2$ 1 / 2] 2] has level $\theta(\theta_2(\Omega_{\omega2}), \Omega)$,
[1 [1 $\bullet\bullet_2$ 1 ●● 2] 2] has level $\theta(\theta_2(\Omega_{\omega2}), \theta_1(\Omega_{\omega2}))$,
[1 [1 $\bullet\bullet_2$ 1 $/_2$ 2] 2] has level $\theta(\theta_2(\Omega_{\omega2})+1)$,
[1 [1 $\bullet\bullet_2$ 1 $\bullet\bullet_2$ 2] 2] has level $\theta(\theta_2(\Omega_{\omega2})2)$,
[1 [1 [2 $\bullet\bullet_3$ 2] 2] 2] has level $\theta(\theta_2(\Omega_{\omega2})\omega)$,
[1 [1 [1 / 2 $\bullet\bullet_3$ 2] 2] 2] has level $\theta(\theta_2(\Omega_{\omega2})\Omega)$,
[1 [1 [1 ●● 2 $\bullet\bullet_3$ 2] 2] 2] has level $\theta(\theta_2(\Omega_{\omega2})\theta_1(\Omega_{\omega2}))$,
[1 [1 [1 $/_2$ 2 $\bullet\bullet_3$ 2] 2] 2] has level $\theta(\theta_2(\Omega_{\omega2})\Omega_2)$,
[1 [1 [1 $\bullet\bullet_2$ 2 $\bullet\bullet_3$ 2] 2] 2] has level $\theta(\theta_2(\Omega_{\omega2})\text{^}2)$,
[1 [1 [1 $/_3$ 2 $\bullet\bullet_3$ 2] 2] 2] has level $\theta(\varepsilon(\theta_2(\Omega_{\omega2})+1))$,
[1 [1 [1 $\bullet_3$ 2 $\bullet\bullet_3$ 2] 2] 2] has level $\theta(\theta_2(\theta_3(\Omega_{\omega}), \theta_2(\Omega_{\omega2})+1))$,
[1 [1 [1 $\bullet\bullet_3$ 3] 2] 2] has level $\theta(\theta_3(\Omega_{\omega2}), 1)$,
[1 [1 [1 [1 $\bullet\bullet_4$ 3] 2] 2] 2] has level $\theta(\theta_4(\Omega_{\omega2}), 1)$,
[1 [1 [1 [1 [1 $\bullet\bullet_5$ 3] 2] 2] 2] 2] has level $\theta(\theta_5(\Omega_{\omega2}), 1)$.

The sequence of separators starting with the last three has limit ordinal $\theta(\theta_{\omega}(\Omega_{\omega2}), 1)$.

The next stage launches the treble forward slash (///), which requires at least three pairs of square brackets around it. The symbol $//_n$ = [1 /// 2 $_n$] in order to mirror $/_n$ = [1 // 2 $_n$], and, just as $/_n$ = [1 $/_{n+1}$ 2] and $//_n$ = [1 $//_{n+1}$ 2], the symbol $///_n$ = [1 $///_{n+1}$ 2]. The (n, 3)-hyperseparator $///_n$ needs a minimum enclosure of n+2 pairs of square brackets. Since I have exhausted // = [1 /// 2] prior to introducing the ●● symbol, $\bullet\bullet_n$ = [1 [2 /// 2] 2 $_n$]. The symbol [2 [2 /// 2] 2] comes next in the sequence after ●●.

[1 [2 [2 /// 2] 2] 2] has level $\theta(\theta_{\omega}(\Omega_{\omega2}), 1)$,
[1 [3 [2 /// 2] 2] 2] has level $\theta(\theta_{\omega}(\Omega_{\omega2}), 2)$,
[1 [1 / 2 [2 /// 2] 2] 2] has level $\theta(\theta_{\omega}(\Omega_{\omega2}), \Omega)$,
[1 [1 ●● 2 [2 /// 2] 2] 2] has level $\theta(\theta_{\omega}(\Omega_{\omega2}), \theta_1(\Omega_{\omega2}))$,
[1 [1 [1 ●● 2 [2 /// 2] 2] 2 [2 /// 2] 2] 2] has level $\theta(\theta_{\omega}(\Omega_{\omega2}), \theta_1(\Omega_{\omega2}, \theta_1(\Omega_{\omega2})))$,
[1 [1 $/_2$ 2 [2 /// 2] 2] 2] has level $\theta(\theta_{\omega}(\Omega_{\omega2}), \Omega_2)$,
[1 [1 $\bullet\bullet_2$ 2 [2 /// 2] 2] 2] has level $\theta(\theta_{\omega}(\Omega_{\omega2}), \theta_2(\Omega_{\omega2}))$,
[1 [1 [1 $\bullet\bullet_2$ 2 [2 /// 2] 2 $_2$] 2 [2 /// 2] 2] 2] has level $\theta(\theta_{\omega}(\Omega_{\omega2}), \theta_2(\Omega_{\omega2}, \theta_2(\Omega_{\omega2})))$,
[1 [1 [1 $/_3$ 2 [2 /// 2] 2 $_2$] 2 [2 /// 2] 2] 2] has level $\theta(\theta_{\omega}(\Omega_{\omega2}), \Omega_3)$,
[1 [1 [1 $\bullet\bullet_3$ 2 [2 /// 2] 2 $_2$] 2 [2 /// 2] 2] 2] has level $\theta(\theta_{\omega}(\Omega_{\omega2}), \theta_3(\Omega_{\omega2}))$,

[1 [1 [1 [1 $/_4$ 2 [2 /// 2] 2 $_3$] 2 [2 /// 2] 2 $_2$] 2 [2 /// 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2}), \Omega_4)$,
[1 [1 [1 [1 [1 $/_5$ 2 [2 /// 2] 2 $_4$] 2 [2 /// 2] 2 $_3$] 2 [2 /// 2] 2 $_2$] 2 [2 /// 2] 2] 2] has level
$\theta(\theta_\omega(\Omega_{\omega2}), \Omega_5)$,
[1 [1 // 2 [2 /// 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})+1)$,
[1 [1 // 1 // 2 [2 /// 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})+\Omega_\omega)$,
[1 [1 [1 / 2 $//_2$ 2] 2 [2 /// 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})+\Omega_\omega\hat{}\Omega)$,
[1 [1 [1 // 2 $//_2$ 2] 2 [2 /// 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})+\Omega_\omega\hat{}\Omega_\omega)$,
[1 [1 [1 $//_2$ 3] 2 [2 /// 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})+\varepsilon(\Omega_\omega+1)) = \theta(\theta_\omega(\Omega_{\omega2})+\theta_\omega(1))$,
[1 [1 [1 [1 // 2 $//_3$ 2] 2] 2 [2 /// 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})+\theta_\omega(\Omega_\omega))$,
[1 [1 [1 [1 $//_2$ 2 $//_3$ 2] 2] 2 [2 /// 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})+\theta_\omega(\Omega_{\omega+1}))$,
[1 [1 [1 [1 [1 $//_3$ 2 $//_4$ 2] 2] 2] 2 [2 /// 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})+\theta_\omega(\Omega_{\omega+2}))$,
[1 [1 [2 /// 2] 3] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})2)$,
[1 [1 [2 /// 2] 1 / 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})\Omega)$,
[1 [1 [2 /// 2] 1 // 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})\Omega_\omega)$,
[1 [1 [2 /// 2] 1 [1 // 2 $//_2$ 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})(\Omega_\omega\hat{}\Omega_\omega))$,
[1 [1 [2 /// 2] 1 [1 $//_2$ 3] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})\varepsilon(\Omega_\omega+1)) = \theta(\theta_\omega(\Omega_{\omega2})\theta_\omega(1))$,
[1 [1 [2 /// 2] 1 [1 [1 // 2 $//_3$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})\theta_\omega(\Omega_\omega))$,
[1 [1 [2 /// 2] 1 [1 [1 $//_2$ 2 $//_3$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})\theta_\omega(\Omega_{\omega+1}))$,
[1 [1 [2 /// 2] 1 [1 [1 [1 $//_3$ 2 $//_4$ 2] 2] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})\theta_\omega(\Omega_{\omega+2}))$,
[1 [1 [2 /// 2] 1 [2 /// 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})\hat{}2)$,
[1 [1 [2 [2 /// 2 $_2$] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})\hat{}\omega)$,
[1 [1 [1 / 2 [2 /// 2 $_2$] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})\hat{}\Omega)$,
[1 [1 [1 // 2 [2 /// 2 $_2$] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})\hat{}\Omega_\omega)$,
[1 [1 [1 [2 /// 2] 2 [2 /// 2 $_2$] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})\hat{}\theta_\omega(\Omega_{\omega2}))$,
[1 [1 [1 $//_2$ 2 [2 /// 2 $_2$] 2] 2] 2] has level $\theta(\varepsilon(\theta_\omega(\Omega_{\omega2})+1))$,
[1 [1 [1 [1 // 2 $//_3$ 2] 2 [2 /// 2 $_2$] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_\omega, \theta_\omega(\Omega_{\omega2})+1))$,
[1 [1 [1 [1 $//_2$ 2 $//_3$ 2] 2 [2 /// 2 $_2$] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega+1}, \theta_\omega(\Omega_{\omega2})+1))$,
[1 [1 [1 [1 [1 $//_3$ 2 $//_4$ 2] 2] 2 [2 /// 2 $_2$] 2] 2] 2] has level $\theta(\theta_\omega(\theta_{\omega+1}(\Omega_{\omega+2}), \theta_\omega(\Omega_{\omega2})+1))$,
[1 [1 [1 [1 [1 [1 $//_4$ 2 $//_5$ 2] 2] 2] 2 [2 /// 2 $_2$] 2] 2] 2] has level $\theta(\theta_\omega(\theta_{\omega+1}(\Omega_{\omega+3}), \theta_\omega(\Omega_{\omega2})+1))$,
[1 [1 [1 [2 /// 2 $_2$] 3] 2] 2] has level $\theta(\theta_{\omega+1}(\Omega_{\omega2}), 1) = \theta(\theta_\omega(\theta_{\omega+1}(\Omega_{\omega2}), 1))$
($\theta_\omega(\theta_{\omega+1}(\Omega_{\omega2}), 1)$ is the limit of $\theta_\omega(\alpha, \theta_\omega(\theta_{\omega+1}(\Omega_{\omega2}))+1)$ as $\alpha \to \theta_{\omega+1}(\Omega_{\omega2})$),
[1 [1 [1 [2 /// 2 $_2$] 1 / 2] 2] 2] has level $\theta(\theta_{\omega+1}(\Omega_{\omega2}), \Omega)$,
[1 [1 [1 [2 /// 2 $_2$] 1 // 2] 2] 2] has level $\theta(\theta_{\omega+1}(\Omega_{\omega2}), \Omega_\omega)$,
[1 [1 [1 [2 /// 2 $_2$] 1 [2 /// 2] 2] 2] 2] has level $\theta(\theta_{\omega+1}(\Omega_{\omega2}), \theta_\omega(\Omega_{\omega2}))$,
[1 [1 [1 [2 /// 2 $_2$] 1 $//_2$ 2] 2] 2] has level $\theta(\theta_{\omega+1}(\Omega_{\omega2})+1)$,
[1 [1 [1 [2 /// 2 $_2$] 1 [2 /// 2 $_2$] 2] 2] 2] has level $\theta(\theta_{\omega+1}(\Omega_{\omega2})2)$,
[1 [1 [1 [1 / 2 [2 /// 2 $_3$] 2] 2] 2] 2] has level $\theta(\theta_{\omega+1}(\Omega_{\omega2})\Omega)$,
[1 [1 [1 [1 // 2 [2 /// 2 $_3$] 2] 2] 2] 2] has level $\theta(\theta_{\omega+1}(\Omega_{\omega2})\Omega_\omega)$,
[1 [1 [1 [1 [2 /// 2] 2 [2 /// 2 $_3$] 2] 2] 2] 2] has level $\theta(\theta_{\omega+1}(\Omega_{\omega2})\theta_\omega(\Omega_{\omega2}))$,
[1 [1 [1 [1 $//_2$ 2 [2 /// 2 $_3$] 2] 2] 2] 2] has level $\theta(\theta_{\omega+1}(\Omega_{\omega2})\Omega_{\omega+1})$,
[1 [1 [1 [1 [2 /// 2 $_2$] 2 [2 /// 2 $_3$] 2] 2] 2] 2] has level $\theta(\theta_{\omega+1}(\Omega_{\omega2})\hat{}2)$,
[1 [1 [1 [1 $//_3$ 2 [2 /// 2 $_3$] 2] 2] 2] 2] has level $\theta(\varepsilon(\theta_{\omega+1}(\Omega_{\omega2})+1))$,
[1 [1 [1 [1 [1 $//_3$ 2 $//_4$ 2] 2 [2 /// 2 $_3$] 2] 2] 2] 2] has level $\theta(\theta_{\omega+1}(\Omega_{\omega+2}, \theta_{\omega+1}(\Omega_{\omega2})+1))$,
[1 [1 [1 [1 [1 [1 $//_4$ 2 $//_5$ 2] 2] 2 [2 /// 2 $_3$] 2] 2] 2] 2] has level $\theta(\theta_{\omega+1}(\theta_{\omega+2}(\Omega_{\omega+3}), \theta_{\omega+1}(\Omega_{\omega2})+1))$,
[1 [1 [1 [1 [2 /// 2 $_3$] 3] 2] 2] 2] has level $\theta(\theta_{\omega+2}(\Omega_{\omega2}), 1) = \theta(\theta_{\omega+1}(\theta_{\omega+2}(\Omega_{\omega2}), 1))$
($\theta_{\omega+1}(\theta_{\omega+2}(\Omega_{\omega2}), 1)$ is limit of $\theta_{\omega+1}(\alpha, \theta_{\omega+1}(\theta_{\omega+2}(\Omega_{\omega2}))+1)$ as $\alpha \to \theta_{\omega+2}(\Omega_{\omega2})$),
[1 [1 [1 [1 [2 /// 2 $_3$] 1 $//_3$ 2] 2] 2] 2] has level $\theta(\theta_{\omega+2}(\Omega_{\omega2})+1)$,
[1 [1 [1 [1 [2 /// 2 $_3$] 1 [2 /// 2 $_3$] 2] 2] 2] 2] has level $\theta(\theta_{\omega+2}(\Omega_{\omega2})2)$,
[1 [1 [1 [1 [1 // 2 [2 /// 2 $_4$] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega+2}(\Omega_{\omega2})\Omega_\omega)$,

[1 [1 [1 [1 [1 [2 /// 2 $_3$] 2 [2 /// 2 $_4$] 2] 2] 2] 2] 2] has level θ($\theta_{\omega+2}$($\Omega_{\omega2}$)^2),
[1 [1 [1 [1 [1 //$_4$ 2 [2 /// 2 $_4$] 2] 2] 2] 2] 2] has level θ(ε($\theta_{\omega+2}$($\Omega_{\omega2}$)+1)),
[1 [1 [1 [1 [1 [2 /// 2 $_4$] 3] 2] 2] 2] 2] has level θ($\theta_{\omega+3}$($\Omega_{\omega2}$), 1),
[1 [1 [1 [1 [1 [1 [2 /// 2 $_5$] 3] 2] 2] 2] 2] 2] has level θ($\theta_{\omega+4}$($\Omega_{\omega2}$), 1),
[1 [1 [1 [1 [1 [1 [1 [2 /// 2 $_6$] 3] 2] 2] 2] 2] 2] 2] has level θ($\theta_{\omega+5}$($\Omega_{\omega2}$), 1).

The limit ordinal of θ($\theta_{\omega+n}$($\Omega_{\omega2}$), 1) as n → ω is θ($\Omega_{\omega2}$, 1).

Now, the square bracket layer directly containing the treble slash can be changed. The separators continue as follows:

[1 [1 [3 /// 2] 2] 2] has level θ($\Omega_{\omega2}$, 1),
[1 [1 [4 /// 2] 2] 2] has level θ($\Omega_{\omega2}$, 2),
[1 [1 [1 / 2 /// 2] 2] 2] has level θ($\Omega_{\omega2}$, Ω),
[1 [1 [1 /$_2$ 2 /// 2] 2] 2] has level θ($\Omega_{\omega2}$, Ω_2),
[1 [1 [1 [1 [1 /$_3$ 2 /// 2] 2 $_2$] 2 /// 2] 2] 2] has level θ($\Omega_{\omega2}$, Ω_3),
[1 [1 [1 [1 [1 [1 [1 /$_4$ 2 /// 2] 2 $_3$] 2 /// 2] 2 $_2$] 2 /// 2] 2] 2] has level θ($\Omega_{\omega2}$, Ω_4),
[1 [1 [1 // 2 /// 2] 2] 2] has level θ($\Omega_{\omega2}$, Ω_ω),
[1 [1 [1 [2 /// 2] 2 /// 2] 2] 2] has level θ($\Omega_{\omega2}$, θ_ω($\Omega_{\omega2}$)),
[1 [1 [1 [1 [2 /// 2] 2 /// 2] 2 /// 2] 2] 2] has level θ($\Omega_{\omega2}$, θ_ω($\Omega_{\omega2}$, θ_ω($\Omega_{\omega2}$))),
[1 [1 [1 //$_2$ 2 /// 2] 2] 2] has level θ($\Omega_{\omega2}$, $\Omega_{\omega+1}$),
[1 [1 [1 [2 /// 2 $_2$] 2 /// 2] 2] 2] has level θ($\Omega_{\omega2}$, $\theta_{\omega+1}$($\Omega_{\omega2}$)),
[1 [1 [1 [1 //$_3$ 2 /// 2 $_2$] 2 /// 2] 2] 2] has level θ($\Omega_{\omega2}$, $\Omega_{\omega+2}$),
[1 [1 [1 [1 [1 //$_4$ 2 /// 2 $_3$] 2 /// 2 $_2$] 2 /// 2] 2] 2] has level θ($\Omega_{\omega2}$, $\Omega_{\omega+3}$),
[1 [1 [1 /// 3] 2] 2] has level θ($\Omega_{\omega2}$+1),
[1 [1 [1 /// 1 / 2] 2] 2] has level θ($\Omega_{\omega2}$+Ω),
[1 [1 [1 /// 1 /$_2$ 2] 2] 2] has level θ($\Omega_{\omega2}$+Ω_2),
[1 [1 [1 /// 1 [1 [1 /// 1 /$_3$ 2] 2 $_2$] 2] 2] 2] has level θ($\Omega_{\omega2}$+Ω_3),
[1 [1 [1 /// 1 // 2] 2] 2] has level θ($\Omega_{\omega2}$+Ω_ω),
[1 [1 [1 /// 1 //$_2$ 2] 2] 2] has level θ($\Omega_{\omega2}$+$\Omega_{\omega+1}$),
[1 [1 [1 /// 1 [1 /// 1 //$_3$ 2 $_2$] 2] 2] 2] has level θ($\Omega_{\omega2}$+$\Omega_{\omega+2}$),
[1 [1 [1 /// 1 /// 2] 2] 2] has level θ($\Omega_{\omega2}$2),
[1 [1 [1 /// 1 /// 1 / 2] 2] 2] has level θ($\Omega_{\omega2}$Ω),
[1 [1 [1 /// 1 /// 1 // 2] 2] 2] has level θ($\Omega_{\omega2}\Omega_\omega$),
[1 [1 [1 /// 1 /// 1 /// 2] 2] 2] has level θ($\Omega_{\omega2}$^2),
[1 [1 [1 /// 1 /// 1 /// 1 /// 2] 2] 2] has level θ($\Omega_{\omega2}$^3),
[1 [1 [1 [2 ///$_2$ 2] 2] 2] 2] has level θ($\Omega_{\omega2}$^ω),
[1 [1 [1 [1 / 2 ///$_2$ 2] 2] 2] 2] has level θ($\Omega_{\omega2}$^Ω),
[1 [1 [1 [1 /// 2 ///$_2$ 2] 2] 2] 2] has level θ($\Omega_{\omega2}$^$\Omega_{\omega2}$),
[1 [1 [1 [1 /// 1 /// 2 ///$_2$ 2] 2] 2] 2] has level θ($\Omega_{\omega2}$^$\Omega_{\omega2}$^$\Omega_{\omega2}$),
[1 [1 [1 [1 [1 /// 2 ///$_2$ 2] 2 ///$_2$ 2] 2] 2] 2] has level θ($\Omega_{\omega2}$^$\Omega_{\omega2}$^$\Omega_{\omega2}$^$\Omega_{\omega2}$),
[1 [1 [1 [1 ///$_2$ 3] 2] 2] 2] has level θ(ε($\Omega_{\omega2}$+1)) = θ($\theta_{\omega2}$(1)),
[1 [1 [1 [1 ///$_2$ 4] 2] 2] 2] has level θ(ε($\Omega_{\omega2}$+2)) = θ($\theta_{\omega2}$(1, 1)),
[1 [1 [1 [1 ///$_2$ 1 / 2] 2] 2] 2] has level θ(ε($\Omega_{\omega2}$+Ω)) = θ($\theta_{\omega2}$(1, Ω)),
[1 [1 [1 [1 ///$_2$ 1 /// 2] 2] 2] 2] has level θ(ε($\Omega_{\omega2}$2)) = θ($\theta_{\omega2}$(1, $\Omega_{\omega2}$)),
[1 [1 [1 [1 ///$_2$ 1 [1 ///$_2$ 3] 2] 2] 2] 2] has level θ(ε(ε($\Omega_{\omega2}$+1))),
[1 [1 [1 [1 ///$_2$ 1 ///$_2$ 2] 2] 2] 2] has level θ(ζ($\Omega_{\omega2}$+1)) = θ($\theta_{\omega2}$(2)),
[1 [1 [1 [1 [2 ///$_3$ 2] 2] 2] 2] 2] has level θ($\theta_{\omega2}$(ω)),
[1 [1 [1 [1 [1 / 2 ///$_3$ 2] 2] 2] 2] 2] has level θ($\theta_{\omega2}$(Ω)),
[1 [1 [1 [1 [1 /// 2 ///$_3$ 2] 2] 2] 2] 2] has level θ($\theta_{\omega2}$($\Omega_{\omega2}$)),
[1 [1 [1 [1 [1 [1 ///$_2$ 3] 2 ///$_3$ 2] 2] 2] 2] 2] has level θ($\theta_{\omega2}$($\theta_{\omega2}$(1))),

[1 [1 [1 [1 [1 [1 [1 [1 $///_2$ 3] 2 $///_3$ 2] 2] 2 $///_3$ 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega2}(\theta_{\omega2}(\theta_{\omega2}(1))))$,
[1 [1 [1 [1 [1 $///_2$ 2 $///_3$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega2+1}) = \theta(\theta_{\omega2}(\Omega_{\omega2+1}))$,
[1 [1 [1 [1 [1 $///_3$ 3] 2] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega2+1}+1)) = \theta(\theta_{\omega2+1}(1))$,
[1 [1 [1 [1 [1 $///_3$ 1 $///_3$ 2] 2] 2] 2] 2] has level $\theta(\zeta(\Omega_{\omega2+1}+1)) = \theta(\theta_{\omega2+1}(2))$,
[1 [1 [1 [1 [1 [2 $///_4$ 2] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega2+1}(\omega))$,
[1 [1 [1 [1 [1 [1 / 2 $///_4$ 2] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega2+1}(\Omega))$,
[1 [1 [1 [1 [1 [1 /// 2 $///_4$ 2] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega2+1}(\Omega_{\omega2}))$,
[1 [1 [1 [1 [1 [1 $///_2$ 2 $///_4$ 2] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega2+1}(\Omega_{\omega2+1}))$,
[1 [1 [1 [1 [1 [1 $///_3$ 2 $///_4$ 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega2+2})$,
[1 [1 [1 [1 [1 [1 $///_4$ 3] 2] 2] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega2+2}+1)) = \theta(\theta_{\omega2+2}(1))$,
[1 [1 [1 [1 [1 [1 $///_4$ 1 $///_4$ 2] 2] 2] 2] 2] 2] has level $\theta(\zeta(\Omega_{\omega2+2}+1)) = \theta(\theta_{\omega2+2}(2))$,
[1 [1 [1 [1 [1 [1 [2 $///_5$ 2] 2] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega2+2}(\omega))$,
[1 [1 [1 [1 [1 [1 [1 $///_4$ 2 $///_5$ 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega2+3})$,
[1 [1 [1 [1 [1 [1 [1 [1 $///_5$ 2 $///_6$ 2] 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega2+4})$,
[1 [1 [1 [1 [1 [1 [1 [1 [1 $///_6$ 2 $///_7$ 2] 2] 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega2+5})$.

The sequence of separators starting with the last three has limit ordinal $\theta(\Omega_{\omega3})$. This mirrors the pattern on pages 14-15 with double slashes apart from the extra pair of square brackets in the separators and the addition of ω in the Ω and θ function subscripts.

The initial part of Rule A5 is modified to take account of treble slash symbols. Within $[A_{1,p_1}]$, take x_1 to be the highest subscript to a single slash or closed square bracket containing a (1, 2)-hyperseparator as the highest hyperseparator in its ‘base layer’, x_2 to be the highest subscript to a double slash or closed square bracket containing a (1, 3)-hyperseparator as the highest hyperseparator in its ‘base layer’ and x_3 to be the highest subscript to a treble slash (these values are 0 when the related symbols do not exist). The x variable is set as follows:

$x = \omega2+x_3-1$ $(x_3 > 0)$
$= \omega+x_2-1$ $(x_2 > 0, x_3 = 0)$
$= x_1$ $(x_2 = 0, x_3 = 0)$.

The number of t-counters required is $x_1+x_2+x_3$ (x_1 below the ωth t-counter, x_2 from the ωth t-counter up to but not including the (ω2)th t-counter and x_3 from the (ω2)th t-counter onwards). The maximum value of this, or of x, is now ω3.

A new Rule A5a3 (separator $[A_{i,p_i}] = ///_m$, where m ≥ 1) would need to be created, which is exactly identical to Rule A5a2 except that the $//_m$ in the $R_{n,i}$ string becomes $///_m$ and $s = i-t_{\omega2+m-1}$. The $\#_H$ string in Rule A5a* contains either a (1, 2)- or (1, 3)-hyperseparator as the highest hyperseparator in its ‘base layer’.

The $\theta(\Omega_{\omega2})$ level separator on page 16 is verified using the revised Rule A5a*:

{a, b [1 ●● 2] 2} = {a ‹0 [1 [2 /// 2] 2] 2› b}
= {a ‹S_1› b}.

Since $p_1 = 1$, $c_1 = 2$, $\#_1 = \#^* =$ ‘’ (blank),
$[A_{1,1}]$ = [1 [2 /// 2] 2] (1-hyperseparator),
$p_2 = 1$, $c_2 = 2$, $\#_2 =$ ‘’ (blank),
$[A_{2,1}]$ = [2 /// 2] ((1, 2)-hyperseparator or ω-hyperseparator),

by Rule A5b,
S_1 = ‘b ‹S_2› b’,

and by Rule A5a* (d = 2, $\#_H$ = ‘/// 2’, m = 1),
S_2 = ‘R_b’,
R_n = ‘b ‹R_{n-1}› b’,

R_1 = ‘b [1 /// 2 $_b$] b’
= ‘b $//_b$ b’.

At this stage, it is better to rewrite $//_n$ as $/_{n,2}$ and $///_n$ as $/_{n,3}$. The symbols // and /// can be rewritten as $/_{1,2}$ and $/_{1,3}$ respectively; $/_{n,1}$ is $/_n$ (remove trailing 1’s). There are two directions of travel for the generalised (m, n)-hyperseparator double subscript slash symbol ($/_{m,n}$), since

$/_{m,n}$ = [1 $/_{m+1,n}$ 2] = [1 $/_{1,n+1}$ 2 $_m$]   (both [1 $/_{m+1,n}$ 2] and [1 $/_{1,n+1}$ 2 $_m$] ‘drop down’ to $/_{m,n}$).

$/_{m,n}$ requires a minimum of m+n-1 pairs of square brackets around it in order to be used in an array.

With k pairs of square brackets (k ≥ 2), the $\theta(\Omega_{\omega(k-1)})$ level separator

{a, b [1 [1 [... [1 [2 $/_{1,k}$ 2] 2] ...] 2] 2] 2} = {a ‹0 [1 [... [1 [2 $/_{1,k}$ 2] 2] ...] 2] 2› b}
= {a ‹b ‹b ‹ ... ‹b ‹b $/_{b,k-1}$ b› b› ... › b› b› b}
(with b+k-2 pairs of angle brackets).

The most significant separators from $\theta(\Omega_\omega)$ to $\theta(\Omega_{\omega2})$ ordinal level are:-

[1 [2 $/_{1,2}$ 2] 2] has level $\theta(\Omega_\omega)$,
[1 [2 $/_{1,2}$ 2] 3] has level $\theta(\theta_1(\Omega_\omega), 1)$,
[1 [2 $/_{1,2}$ 2] 1 / 2] has level $\theta(\theta_1(\Omega_\omega)+1)$,
[1 [2 $/_{1,2}$ 2] 1 [2 $/_{1,2}$ 2] 2] has level $\theta(\theta_1(\Omega_\omega)2)$,
[1 [2 [2 $/_{1,2}$ 2 $_2$] 2] 2] has level $\theta(\theta_1(\Omega_\omega)\omega)$,
[1 [1 [2 $/_{1,2}$ 2] 2 [2 $/_{1,2}$ 2 $_2$] 2] 2] has level $\theta(\theta_1(\Omega_\omega)$^2),
[1 [1 $/_2$ 2 [2 $/_{1,2}$ 2 $_2$] 2] 2] has level $\theta(\varepsilon(\theta_1(\Omega_\omega)+1))$,
[1 [1 [2 $/_{1,2}$ 2 $_2$] 3] 2] has level $\theta(\theta_2(\Omega_\omega), 1)$,
[1 [1 [1 [2 $/_{1,2}$ 2 $_3$] 3] 2] 2] has level $\theta(\theta_3(\Omega_\omega), 1)$,
[1 [3 $/_{1,2}$ 2] 2] has level $\theta(\Omega_\omega, 1)$,
[1 [1 / 2 $/_{1,2}$ 2] 2] has level $\theta(\Omega_\omega, \Omega)$,
[1 [1 [2 $/_{1,2}$ 2] 2 $/_{1,2}$ 2] 2] has level $\theta(\Omega_\omega, \theta_1(\Omega_\omega))$,
[1 [1 $/_2$ 2 $/_{1,2}$ 2] 2] has level $\theta(\Omega_\omega, \Omega_2)$,
[1 [1 [1 $/_3$ 2 $/_{1,2}$ 2 $_2$] 2 $/_{1,2}$ 2] 2] has level $\theta(\Omega_\omega, \Omega_3)$,
[1 [1 $/_{1,2}$ 3] 2] has level $\theta(\Omega_\omega+1)$,
[1 [1 $/_{1,2}$ 1 $/_{1,2}$ 2] 2] has level $\theta(\Omega_\omega 2)$,
[1 [1 $/_{1,2}$ 1 $/_{1,2}$ 1 $/_{1,2}$ 2] 2] has level $\theta(\Omega_\omega$^2),
[1 [1 [2 $/_{2,2}$ 2] 2] 2] has level $\theta(\Omega_\omega$^ω),
[1 [1 [1 $/_{1,2}$ 2 $/_{2,2}$ 2] 2] 2] has level $\theta(\Omega_\omega$^$\Omega_\omega)$,
[1 [1 [1 $/_{2,2}$ 3] 2] 2] has level $\theta(\varepsilon(\Omega_\omega+1))$,
[1 [1 [1 $/_{2,2}$ 1 $/_{2,2}$ 2] 2] 2] has level $\theta(\zeta(\Omega_\omega+1))$,
[1 [1 [1 [2 $/_{3,2}$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\omega))$,
[1 [1 [1 [1 / 2 $/_{3,2}$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\Omega))$,
[1 [1 [1 [1 $/_{1,2}$ 2 $/_{3,2}$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_\omega))$,
[1 [1 [1 [1 $/_{2,2}$ 2 $/_{3,2}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega+1})$,
[1 [1 [1 [1 $/_{3,2}$ 3] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega+1}+1))$,
[1 [1 [1 [1 $/_{3,2}$ 1 $/_{3,2}$ 2] 2] 2] 2] has level $\theta(\zeta(\Omega_{\omega+1}+1))$,
[1 [1 [1 [1 [2 $/_{4,2}$ 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega+1}(\omega))$,
[1 [1 [1 [1 [1 $/_{3,2}$ 2 $/_{4,2}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega+2})$,
[1 [1 [1 [1 [1 [1 $/_{4,2}$ 2 $/_{5,2}$ 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega+3})$.

Significant separators from $\theta(\Omega_{\omega2})$ to $\theta(\Omega_{\omega3})$ level are:-

[1 [1 [2 $/_{1,3}$ 2] 2] 2] has level $\theta(\Omega_{\omega2})$,
[1 [1 [2 $/_{1,3}$ 2] 2] 3] has level $\theta(\theta_1(\Omega_{\omega2}), 1)$,

[1 [1 [1 [2 /$_{1,3}$ 2] 2 $_2$] 3] 2] has level $\theta(\theta_2(\Omega_{\omega2}), 1)$,
[1 [1 [1 [1 [2 /$_{1,3}$ 2] 2 $_3$] 3] 2] 2] has level $\theta(\theta_3(\Omega_{\omega2}), 1)$,
[1 [2 [2 /$_{1,3}$ 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2}), 1)$,
[1 [1 / 2 [2 /$_{1,3}$ 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2}), \Omega)$,
[1 [1 /$_2$ 2 [2 /$_{1,3}$ 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2}), \Omega_2)$,
[1 [1 [1 /$_3$ 2 [2 /$_{1,3}$ 2] 2 $_2$] 2 [2 /$_{1,3}$ 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2}), \Omega_3)$,
[1 [1 /$_{1,2}$ 2 [2 /$_{1,3}$ 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})+1)$,
[1 [1 [2 /$_{1,3}$ 2] 3] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})2)$,
[1 [1 [2 /$_{1,3}$ 2] 1 [2 /$_{1,3}$ 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})\char`^2)$,
[1 [1 [1 [2 /$_{1,3}$ 2] 2 [2 /$_{1,3}$ 2 $_2$] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega2})\char`^\theta_\omega(\Omega_{\omega2}))$,
[1 [1 [1 /$_{2,2}$ 2 [2 /$_{1,3}$ 2 $_2$] 2] 2] 2] has level $\theta(\varepsilon(\theta_\omega(\Omega_{\omega2})+1))$,
[1 [1 [1 [2 /$_{1,3}$ 2 $_2$] 3] 2] 2] has level $\theta(\theta_{\omega+1}(\Omega_{\omega2}), 1)$,
[1 [1 [1 [1 [2 /$_{1,3}$ 2 $_3$] 3] 2] 2] 2] has level $\theta(\theta_{\omega+2}(\Omega_{\omega2}), 1)$,
[1 [1 [1 [1 [1 [2 /$_{1,3}$ 2 $_4$] 3] 2] 2] 2] 2] has level $\theta(\theta_{\omega+3}(\Omega_{\omega2}), 1)$,
[1 [1 [3 /$_{1,3}$ 2] 2] 2] has level $\theta(\Omega_{\omega2}, 1)$,
[1 [1 [1 / 2 /$_{1,3}$ 2] 2] 2] has level $\theta(\Omega_{\omega2}, \Omega)$,
[1 [1 [1 /$_{1,2}$ 2 /$_{1,3}$ 2] 2] 2] has level $\theta(\Omega_{\omega2}, \Omega_\omega)$,
[1 [1 [1 /$_{2,2}$ 2 /$_{1,3}$ 2] 2] 2] has level $\theta(\Omega_{\omega2}, \Omega_{\omega+1})$,
[1 [1 [1 [1 /$_{3,2}$ 2 /$_{1,3}$ 2 $_2$] 2 /$_{1,3}$ 2] 2] 2] has level $\theta(\Omega_{\omega2}, \Omega_{\omega+2})$,
[1 [1 [1 /$_{1,3}$ 3] 2] 2] has level $\theta(\Omega_{\omega2}+1)$,
[1 [1 [1 /$_{1,3}$ 1 /$_{1,3}$ 2] 2] 2] has level $\theta(\Omega_{\omega2}2)$,
[1 [1 [1 /$_{1,3}$ 1 /$_{1,3}$ 1 /$_{1,3}$ 2] 2] 2] has level $\theta(\Omega_{\omega2}\char`^2)$,
[1 [1 [1 [2 /$_{2,3}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega2}\char`^\omega)$,
[1 [1 [1 [1 /$_{1,3}$ 2 /$_{2,3}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega2}\char`^\Omega_{\omega2})$,
[1 [1 [1 [1 /$_{2,3}$ 3] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega2}+1))$,
[1 [1 [1 [1 /$_{2,3}$ 1 /$_{2,3}$ 2] 2] 2] 2] has level $\theta(\zeta(\Omega_{\omega2}+1))$,
[1 [1 [1 [1 [2 /$_{3,3}$ 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega2}(\omega))$,
[1 [1 [1 [1 [1 / 2 /$_{3,3}$ 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega2}(\Omega))$,
[1 [1 [1 [1 [1 /$_{1,3}$ 2 /$_{3,3}$ 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega2}(\Omega_{\omega2}))$,
[1 [1 [1 [1 [1 /$_{2,3}$ 2 /$_{3,3}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega2+1})$,
[1 [1 [1 [1 [1 /$_{3,3}$ 3] 2] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega2+1}+1))$,
[1 [1 [1 [1 [1 /$_{3,3}$ 1 /$_{3,3}$ 2] 2] 2] 2] 2] has level $\theta(\zeta(\Omega_{\omega2+1}+1))$,
[1 [1 [1 [1 [1 [2 /$_{4,3}$ 2] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega2+1}(\omega))$,
[1 [1 [1 [1 [1 [1 /$_{3,3}$ 2 /$_{4,3}$ 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega2+2})$,
[1 [1 [1 [1 [1 [1 [1 /$_{4,3}$ 2 /$_{5,3}$ 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega2+3})$.

Continuing this sequence, I obtain
[1 [1 [1 [2 /$_{1,4}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega3})$,
[1 [1 [1 [2 /$_{1,4}$ 2] 2] 2] 3] has level $\theta(\theta_1(\Omega_{\omega3}), 1)$,
[1 [1 [1 [1 [2 /$_{1,4}$ 2] 2] 2 $_2$] 3] 2] has level $\theta(\theta_2(\Omega_{\omega3}), 1)$,
[1 [1 [1 [1 [1 [2 /$_{1,4}$ 2] 2] 2 $_3$] 3] 2] 2] has level $\theta(\theta_3(\Omega_{\omega3}), 1)$,
[1 [2 [1 [2 /$_{1,4}$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega3}), 1)$,
[1 [1 / 2 [1 [2 /$_{1,4}$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega3}), \Omega)$,
[1 [1 /$_2$ 2 [1 [2 /$_{1,4}$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega3}), \Omega_2)$,
[1 [1 [1 /$_3$ 2 [1 [2 /$_{1,4}$ 2] 2] 2 $_2$] 2 [1 [2 /$_{1,4}$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega3}), \Omega_3)$,
[1 [1 /$_{1,2}$ 2 [1 [2 /$_{1,4}$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega3})+1)$,
[1 [1 [1 [2 /$_{1,4}$ 2] 2] 3] 2] has level $\theta(\theta_\omega(\Omega_{\omega3})2)$,
[1 [1 [1 [2 /$_{1,4}$ 2] 2] 1 [1 [2 /$_{1,4}$ 2] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega3})\char`^2)$,
[1 [1 [1 [1 [2 /$_{1,4}$ 2] 2] 2 [1 [2 /$_{1,4}$ 2] 2 $_2$] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega3})\char`^\theta_\omega(\Omega_{\omega3}))$,

[1 [1 [1 $/_{2,2}$ 2 [1 [2 $/_{1,4}$ 2] 2_2] 2] 2] 2] has level $\theta(\varepsilon(\theta_\omega(\Omega_{\omega3})+1))$,
[1 [1 [1 [1 [2 $/_{1,4}$ 2] 2_2] 3] 2] 2] has level $\theta(\theta_{\omega+1}(\Omega_{\omega3}), 1)$,
[1 [1 [1 [1 [1 [2 $/_{1,4}$ 2] 2_3] 3] 2] 2] 2] has level $\theta(\theta_{\omega+2}(\Omega_{\omega3}), 1)$,
[1 [1 [1 [1 [1 [1 [2 $/_{1,4}$ 2] 2_4] 3] 2] 2] 2] 2] has level $\theta(\theta_{\omega+3}(\Omega_{\omega3}), 1)$,
[1 [1 [2 [2 $/_{1,4}$ 2] 2] 2] 2] has level $\theta(\theta_{\omega2}(\Omega_{\omega3}), 1)$,
[1 [1 [1 / 2 [2 $/_{1,4}$ 2] 2] 2] 2] has level $\theta(\theta_{\omega2}(\Omega_{\omega3}), \Omega)$,
[1 [1 [1 $/_{1,2}$ 2 [2 $/_{1,4}$ 2] 2] 2] 2] has level $\theta(\theta_{\omega2}(\Omega_{\omega3}), \Omega_\omega)$,
[1 [1 [1 $/_{2,2}$ 2 [2 $/_{1,4}$ 2] 2] 2] 2] has level $\theta(\theta_{\omega2}(\Omega_{\omega3}), \Omega_{\omega+1})$,
[1 [1 [1 [1 $/_{3,2}$ 2 [2 $/_{1,4}$ 2] 2_2] 2 [2 $/_{1,4}$ 2] 2] 2] 2] has level $\theta(\theta_{\omega2}(\Omega_{\omega3}), \Omega_{\omega+2})$,
[1 [1 [1 $/_{1,3}$ 2 [2 $/_{1,4}$ 2] 2] 2] 2] has level $\theta(\theta_{\omega2}(\Omega_{\omega3})+1)$,
[1 [1 [1 [2 $/_{1,4}$ 2] 3] 2] 2] has level $\theta(\theta_{\omega2}(\Omega_{\omega3})2)$,
[1 [1 [1 [2 $/_{1,4}$ 2] 1 [2 $/_{1,4}$ 2] 2] 2] 2] has level $\theta(\theta_{\omega2}(\Omega_{\omega3})$^2),
[1 [1 [1 [1 [2 $/_{1,4}$ 2] 2 [2 $/_{1,4}$ 2_2] 2] 2] 2] 2] has level $\theta(\theta_{\omega2}(\Omega_{\omega3})$^$\theta_{\omega2}(\Omega_{\omega3}))$,
[1 [1 [1 [1 $/_{2,3}$ 2 [2 $/_{1,4}$ 2_2] 2] 2] 2] 2] has level $\theta(\varepsilon(\theta_{\omega2}(\Omega_{\omega3})+1))$,
[1 [1 [1 [1 [2 $/_{1,4}$ 2_2] 3] 2] 2] 2] has level $\theta(\theta_{\omega2+1}(\Omega_{\omega3}), 1)$,
[1 [1 [1 [1 [1 [2 $/_{1,4}$ 2_3] 3] 2] 2] 2] 2] has level $\theta(\theta_{\omega2+2}(\Omega_{\omega3}), 1)$,
[1 [1 [1 [1 [1 [1 [2 $/_{1,4}$ 2_4] 3] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega2+3}(\Omega_{\omega3}), 1)$,
[1 [1 [1 [3 $/_{1,4}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega3}, 1)$,
[1 [1 [1 [1 / 2 $/_{1,4}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega3}, \Omega)$,
[1 [1 [1 [1 $/_{1,2}$ 2 $/_{1,4}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega3}, \Omega_\omega)$,
[1 [1 [1 [1 $/_{1,3}$ 2 $/_{1,4}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega3}, \Omega_{\omega2})$,
[1 [1 [1 [1 $/_{2,3}$ 2 $/_{1,4}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega3}, \Omega_{\omega2+1})$,
[1 [1 [1 [1 [1 $/_{3,3}$ 2 $/_{1,4}$ 2_2] 2 $/_{1,4}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega3}, \Omega_{\omega2+2})$,
[1 [1 [1 [1 $/_{1,4}$ 3] 2] 2] 2] has level $\theta(\Omega_{\omega3}+1)$,
[1 [1 [1 [1 $/_{1,4}$ 1 $/_{1,4}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega3}2)$,
[1 [1 [1 [1 $/_{1,4}$ 1 $/_{1,4}$ 1 $/_{1,4}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega3}$^2),
[1 [1 [1 [1 [2 $/_{2,4}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega3}$^$\omega)$,
[1 [1 [1 [1 [1 $/_{1,4}$ 2 $/_{2,4}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega3}$^$\Omega_{\omega3})$,
[1 [1 [1 [1 [1 $/_{2,4}$ 3] 2] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega3}+1)) = \theta(\theta_{\omega3}(1))$,
[1 [1 [1 [1 [1 $/_{2,4}$ 1 $/_{2,4}$ 2] 2] 2] 2] 2] has level $\theta(\zeta(\Omega_{\omega3}+1)) = \theta(\theta_{\omega3}(2))$,
[1 [1 [1 [1 [1 [2 $/_{3,4}$ 2] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega3}(\omega))$,
[1 [1 [1 [1 [1 [1 / 2 $/_{3,4}$ 2] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega3}(\Omega))$,
[1 [1 [1 [1 [1 [1 $/_{1,4}$ 2 $/_{3,4}$ 2] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega3}(\Omega_{\omega3}))$,
[1 [1 [1 [1 [1 [1 $/_{2,4}$ 2 $/_{3,4}$ 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega3+1})$,
[1 [1 [1 [1 [1 [1 $/_{3,4}$ 3] 2] 2] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega3+1}+1)) = \theta(\theta_{\omega3+1}(1))$,
[1 [1 [1 [1 [1 [1 $/_{3,4}$ 1 $/_{3,4}$ 2] 2] 2] 2] 2] 2] has level $\theta(\zeta(\Omega_{\omega3+1}+1)) = \theta(\theta_{\omega3+1}(2))$,
[1 [1 [1 [1 [1 [1 [2 $/_{4,4}$ 2] 2] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega3+1}(\omega))$,
[1 [1 [1 [1 [1 [1 [1 $/_{3,4}$ 2 $/_{4,4}$ 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega3+2})$,
[1 [1 [1 [1 [1 [1 [1 [1 $/_{4,4}$ 2 $/_{5,4}$ 2] 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega3+3})$.

[1 [1 [1 [1 [2 $/_{1,5}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega4})$,
[1 [2 [1 [1 [2 $/_{1,5}$ 2] 2] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega4}), 1)$,
[1 [1 [2 [1 [2 $/_{1,5}$ 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega2}(\Omega_{\omega4}), 1)$,
[1 [1 [1 [2 [2 $/_{1,5}$ 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega3}(\Omega_{\omega4}), 1)$,
[1 [1 [1 [1 $/_{1,4}$ 2 [2 $/_{1,5}$ 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega3}(\Omega_{\omega4})+1)$,
[1 [1 [1 [1 [2 $/_{1,5}$ 2] 3] 2] 2] 2] has level $\theta(\theta_{\omega3}(\Omega_{\omega4})2)$,
[1 [1 [1 [1 [2 $/_{1,5}$ 2] 1 [2 $/_{1,5}$ 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega3}(\Omega_{\omega4})$^2),
[1 [1 [1 [1 [1 $/_{2,4}$ 2 [2 $/_{1,5}$ 2_2] 2] 2] 2] 2] 2] has level $\theta(\varepsilon(\theta_{\omega3}(\Omega_{\omega4})+1))$,
[1 [1 [1 [1 [1 [2 $/_{1,5}$ 2_2] 3] 2] 2] 2] 2] has level $\theta(\theta_{\omega3+1}(\Omega_{\omega4}), 1)$,

[1 [1 [1 [1 [1 [1 [2 $/_{1,5}$ 2 $_{3}$] 3] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega3+2}(\Omega_{\omega4}), 1)$,
[1 [1 [1 [1 [3 $/_{1,5}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega4}, 1)$,
[1 [1 [1 [1 [1 / 2 $/_{1,5}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega4}, \Omega)$,
[1 [1 [1 [1 [1 $/_{1,2}$ 2 $/_{1,5}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega4}, \Omega_{\omega})$,
[1 [1 [1 [1 [1 $/_{1,3}$ 2 $/_{1,5}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega4}, \Omega_{\omega2})$,
[1 [1 [1 [1 [1 $/_{1,4}$ 2 $/_{1,5}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega4}, \Omega_{\omega3})$,
[1 [1 [1 [1 [1 $/_{1,5}$ 3] 2] 2] 2] 2] has level $\theta(\Omega_{\omega4}+1)$,
[1 [1 [1 [1 [1 $/_{1,5}$ 1 $/_{1,5}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega4}2)$,
[1 [1 [1 [1 [1 [2 $/_{2,5}$ 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega4}\text{^}\omega)$,
[1 [1 [1 [1 [1 [1 $/_{1,5}$ 2 $/_{2,5}$ 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega4}\text{^}\Omega_{\omega4})$,
[1 [1 [1 [1 [1 [1 $/_{2,5}$ 3] 2] 2] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega4}+1)) = \theta(\theta_{\omega4}(1))$,
[1 [1 [1 [1 [1 [1 $/_{2,5}$ 1 $/_{2,5}$ 2] 2] 2] 2] 2] 2] has level $\theta(\zeta(\Omega_{\omega4}+1)) = \theta(\theta_{\omega4}(2))$,
[1 [1 [1 [1 [1 [1 [2 $/_{3,5}$ 2] 2] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega4}(\omega))$,
[1 [1 [1 [1 [1 [1 [1 $/_{2,5}$ 2 $/_{3,5}$ 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega4+1})$,
[1 [1 [1 [1 [1 [1 [1 $/_{3,5}$ 3] 2] 2] 2] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega4+1}+1)) = \theta(\theta_{\omega4+1}(1))$,
[1 [1 [1 [1 [1 [1 [1 [1 $/_{3,5}$ 2 $/_{4,5}$ 2] 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega4+2})$,
[1 [1 [1 [1 [1 [1 [1 [1 [1 $/_{4,5}$ 2 $/_{5,5}$ 2] 2] 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega4+3})$.

[1 [1 [1 [1 [1 [2 $/_{1,6}$ 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega5})$,
[1 [1 [1 [1 [1 [3 $/_{1,6}$ 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega5}, 1)$,
[1 [1 [1 [1 [1 [1 $/_{1,6}$ 3] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega5}+1)$,
[1 [1 [1 [1 [1 [1 $/_{1,6}$ 1 $/_{1,6}$ 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega5}2)$,
[1 [1 [1 [1 [1 [1 [1 $/_{1,6}$ 2 $/_{2,6}$ 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega5}\text{^}\Omega_{\omega5})$,
[1 [1 [1 [1 [1 [1 [1 $/_{2,6}$ 3] 2] 2] 2] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega5}+1)) = \theta(\theta_{\omega5}(1))$,
[1 [1 [1 [1 [1 [1 [1 $/_{2,6}$ 1 $/_{2,6}$ 2] 2] 2] 2] 2] 2] 2] has level $\theta(\zeta(\Omega_{\omega5}+1)) = \theta(\theta_{\omega5}(2))$,
[1 [1 [1 [1 [1 [1 [1 [2 $/_{3,6}$ 2] 2] 2] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega5}(\omega))$,
[1 [1 [1 [1 [1 [1 [1 [1 $/_{2,6}$ 2 $/_{3,6}$ 2] 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega5+1})$,
[1 [1 [1 [1 [1 [1 [1 [1 [1 $/_{3,6}$ 2 $/_{4,6}$ 2] 2] 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega5+2})$,
[1 [1 [1 [1 [1 [1 [2 $/_{1,7}$ 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega6})$,
[1 [1 [1 [1 [1 [1 [1 [2 $/_{1,8}$ 2] 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega7})$,
[1 [1 [1 [1 [1 [1 [1 [1 [2 $/_{1,9}$ 2] 2] 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega8})$.

The sequence of separators starting with the last three has limit ordinal $\theta(\Omega_{\omega\text{^}2})$.

In general, with m+n-1 pairs of square brackets (m ≥ 3, n ≥ 2),

[1 [1 [... [1 [1 $/_{m-1,n}$ 2 $/_{m,n}$ 2] 2] ...] 2] 2] has level $\theta(\Omega_{\omega(n-1)+m-2})$.

With n pairs of square brackets (n ≥ 2),

[1 [1 [... [1 [1 $/_{n-1}$ 2 $/_{n}$ 2] 2] ...] 2] 2] has level $\theta(\Omega_{n-1})$,
[1 [1 [... [1 [2 $/_{1,n}$ 2] 2] ...] 2] 2] has level $\theta(\Omega_{\omega(n-1)})$.

We can deduce that the $/_{m,n}$ symbol is associated with the generation of the $\Omega_{\omega(n-1)+m-1}$ uncountable ordinal (n ≥ 2) or Ω_m uncountable ordinal (n = 1) within the θ function.

An (m, n)-hyperseparator is either a $/_{m,n}$ symbol or contains at least one $/_{m+k,n}$ symbol inside k pairs of square brackets or at least one $/_{1,n+1}$ symbol or (1, n+1)-hyperseparator inside k+1 pairs of square brackets (with the highest of the layers having an m+k subscript), for some value of k – but at every value of k there are no $/_{m+k,n}$ symbols inside fewer than k pairs of square brackets or $/_{1,n+1}$ symbols or (1, n+1)-hyperseparators inside fewer than k+1 pairs of square brackets (with the highest of the layers having an m+k subscript).

Putting it another way, a $/_{m,n}$ symbol is an (m, n)-hyperseparator, $/_{m,n}$ enclosed by k pairs of square brackets is an (m-k, n)-hyperseparator (m > k), $/_{m,n}$ enclosed by m pairs of square brackets (subscripted by k at the bottom) is a (k, n-1)-hyperseparator (n ≥ 2), $/_{m,n}$ enclosed by m pairs of square brackets (with no subscript at the bottom) is a (1, n-1)-hyperseparator (n ≥ 2). A $/_{m,n}$ symbol enclosed by m pairs of square brackets (subscripted by m_2 at the bottom), enclosed by m_2 pairs of square brackets (subscripted by m_3 at the bottom), ... , enclosed by m_{n-1} pairs of square brackets (subscripted by m_n at the bottom), enclosed by m_n pairs of square brackets is a normal separator (0-hyperseparator).

The recursive definition of an (m, n)-hyperseparator is that it is either a $/_{m,n}$ symbol or contains either an (m+1, n)-hyperseparator as the highest hyperseparator in its ‘base layer’ or is subscripted by m and contains a (1, n+1)-hyperseparator as the highest hyperseparator in its ‘base layer’. Square brackets may only have a subscript appended to the inside of the closed bracket when the lowest layer contains a (1, n+1)-hyperseparator as highest hyperseparator for some positive integer n. In order to illustrate this, [X $_m$] is only possible when the hyperseparator of highest order in the bottom layer of the string X is a (1, n+1)-hyperseparator (for some n ≥ 1). This is because bracket subscripts are only created by defining $/_{m,n}$ = [1 $/_{1,n+1}$ 2 $_m$] (for m ≥ 1, n ≥ 1) – the lowest in the family of [X $_m$] separators directly containing a (1, n+1)-hyperseparator; the other members of the family eventually ‘reduce’ to the lowest one, which ‘drops down’ to $/_{m,n}$ by application of Rule A5. Separators such as [1 $/_n$ 2 $_m$] or [1 $/_{k,n+1}$ 2 $_m$] (k ≥ 2) are illegal as they are undefined in the notation.

The system of t-counters now requires an overhaul. Instead of t_α (with α transfinite) counting the number of consecutive α- or higher hyperseparators of [A_{i,p_i}] as i increases, we will have t_r (with r always finite) counting the number of successive hyperlevels above a certain rank. I now define [m_i, m_i*] to be the hyperlevel of the [A_{i,p_i}] separator (with [m_i, m_i*] expressed as a 2-entry linear separator array), i.e. [A_{i,p_i}] is an (m_i, m_i*)-hyperseparator. Rule A5 begins with i = 1 and t_1 = 0. Every time Rule A5b is executed ([m_i, m_i*] ≥ [1]), r is taken to be one more than the number of arrays [m_j, m_j*] (for 1 ≤ j < i) that are ‘less than or equal to’ [m_i, m_i*] (a number between 1 and i), a ‘copy flag’ (f, initially at 0) is set to 1 if [m_j, m_j*] = [m_i, m_i*] for at least one value of 1 ≤ j < i, and the t-counters from t_1 to t_r are increased by 1 (with t_r set to t_{r-1} instead if f = 1), while the t-counters from t_{r+1} to t_i are reset to 0 (and we set a new t-counter, t_{i+1} to 0, which becomes the new t_i when i is incremented by 1). Thus, t_r represents the rth lowest out of the first i separators of [A_{i,p_i}] (lower than the (r+1)th lowest but may be identical to the (r-1)th lowest). When Rule A5a or A5a2 (with $//_m$ replaced by $/_{m,m^*}$ (m* ≥ 2)) is executed, we take r to be one more than the number of arrays [m_j, m_j*] (for 1 ≤ j < i) that are ‘less than’ [m, m*], and set s = i-t_r.

Rules A5a, A5a2 and A5a3 can actually be combined into a revised Rule A5a now. The complete Rule A5 is as follows:-

Rule A5 (Rules A1-4 do not apply, first entry is 0, separator immediately prior to next non-1 entry (c_1) is [A_{1,p_1}]):

‘a ‹ 0 [$A_{1,1}$] 1 [$A_{1,2}$] ... 1 [A_{1,p_1}] c_1 $\#_1$ #* › b’ = ‘a ‹S_1 #*› b’,

where p_1 ≥ 1, each of [$A_{1,j}$] is either a normal separator or 1-hyperseparator, $\#_1$ contains no 2- or higher order hyperseparators or subscript in its base layer and #* is either an empty string or subscript or begins with a 2- or higher order hyperseparator.
Set i to 1 and t_1 to 0 and follow Rules A5a-c (a, a* and c are terminal, b is not).

Rule A5a (separator $[A_{i,p_i}] = /_{m,m^*}$, where $m \geq 1$ and $m^* \geq 1$):
Set r to 1. For each j from 1 to i-1, increment r by 1 when either $m_j^* < m^*$, or $m_j^* = m^*$ and $m_j < m$.

$s = i\text{-}t_r$,
h = EndSub(‘$\#_s$’) (subscript at the end of $\#_s$, this is 1 by default),
$\#^*_s$ = DelEndSub(‘$\#_s$’) (identical to $\#_s$ but with the end subscript (h) deleted),
$\#_{n,s}$ = ‘$\#^*_{s\ h+b-n}$’ ($1 < n < b$, $m = 1$, $m^* \geq 2$),
$\#_{n,k}$ = ‘$\#_k$’ ($1 < n \leq b$, $s \leq k \leq i$, except $k = s$, $m = 1$, $m^* \geq 2$),
S_i = ‘$R_{b,i}$’.

For $1 < n \leq b$ and $s \leq k < i$,

$R_{n,i}$ = ‘$b \langle A_{i,1}{}' \rangle\ b\ [A_{i,1}]\ b \langle A_{i,2}{}' \rangle\ b\ [A_{i,2}] \ldots b \langle A_{i,p_i-1}{}' \rangle\ b\ [A_{i,p_i-1}]\ b \langle R_{n-1,s} \rangle\ b\ \ /_{m,m^*}\ \ c_i\text{-}1\ \#_{n,i}$’,
$R_{n,k}$ = ‘$b \langle A_{k,1}{}' \rangle\ b\ [A_{k,1}]\ b \langle A_{k,2}{}' \rangle\ b\ [A_{k,2}] \ldots b \langle A_{k,p_k-1}{}' \rangle\ b\ [A_{k,p_k-1}]\ b \langle R_{n,k+1} \rangle\ b\ [A_{k,p_k}]\ c_k\text{-}1\ \#_{n,k}$’,
$R_{1,s}$ = ‘0’.

Rule A5a* (separator $[A_{i,p_i}] = [d\ \#_{H\ m}]$, where $d \geq 2$ and $\#_H$ contains a (1, k)-hyperseparator as the highest order hyperseparator in its base layer, where $k \geq 2$):

S_i = ‘$b \langle A_{i,1}{}' \rangle\ b\ [A_{i,1}]\ b \langle A_{i,2}{}' \rangle\ b\ [A_{i,2}] \ldots b \langle A_{i,p_i-1}{}' \rangle\ b\ [A_{i,p_i-1}]\ R_b\ [d\ \#_{H\ m}]\ c_i\text{-}1\ \#_i$’,
R_n = ‘$b \langle R_{n-1} \rangle\ b$’ ($n > 1$),
R_1 = ‘$b\ [d\text{-}1\ \#_{H\ m+b-1}]\ b$’.

Rule A5b (Rule A5a does not apply, separator $[A_{i,p_i}] = [1\ [A_{i+1,1}]\ 1\ [A_{i+1,2}] \ldots 1\ [A_{i+1,p_{i+1}}]\ c_{i+1}\ \#_{i+1}]$, which is an ($m_i$, m_i^*)-hyperseparator, where $p_{i+1} \geq 1$, $c_{i+1} \geq 2$, $m_i \geq 1$ and $m_i^* \geq 1$):

S_i = ‘$b \langle A_{i,1}{}' \rangle\ b\ [A_{i,1}]\ b \langle A_{i,2}{}' \rangle\ b\ [A_{i,2}] \ldots b \langle A_{i,p_i-1}{}' \rangle\ b\ [A_{i,p_i-1}]\ b \langle S_{i+1} \rangle\ b\ [A_{i,p_i}]\ c_i\text{-}1\ \#_i$’.

Set r to 1 and f to 0. For each j from 1 to i-1, increment r by 1 when either $m_j^* < m_i^*$, or $m_j^* = m_i^*$ and $m_j \leq m_i$; and set f to 1 when $m_j^* = m_i^*$ and $m_j = m_i$.
Increment $t_1, t_2, \ldots, t_r$ by 1; set t_r to t_{r-1} if $f = 1$; reset $t_{r+1}, t_{r+2}, \ldots, t_i$ to 0; set t_{i+1} to 0; increment i by 1 (so that $i = t_1+1$) and repeat Rules A5a-c.

Rule A5c (Rules A5a-b do not apply):

S_i = ‘$b \langle A_{i,1}{}' \rangle\ b\ [A_{i,1}]\ b \langle A_{i,2}{}' \rangle\ b\ [A_{i,2}] \ldots b \langle A_{i,p_i}{}' \rangle\ b\ [A_{i,p_i}]\ c_i\text{-}1\ \#_i$’.

The $\theta(\Omega_\omega \hat{}\ \Omega_3)$ level separator on page 11 has a sequence of five $[A_{i,p_i}]$ separators, as follows:

$[A_{1,1}] = [1\ [1\ [1\ [1\ /_3\ 2\ /_{2,2}\ 2]\ 2\ _2]\ 2\ /_{2,2}\ 2]\ 2]$, $[m_1, m_1^*] = [1]$,
$[A_{2,1}] = [1\ [1\ [1\ /_3\ 2\ /_{2,2}\ 2]\ 2\ _2]\ 2\ /_{2,2}\ 2]$, $[m_2, m_2^*] = [1, 2]$,
$[A_{3,1}] = [1\ [1\ /_3\ 2\ /_{2,2}\ 2]\ 2\ _2]$, $[m_3, m_3^*] = [2]$,
$[A_{4,1}] = [1\ /_3\ 2\ /_{2,2}\ 2]$, $[m_4, m_4^*] = [1, 2]$,
$[A_{5,1}] = /_3$, $[m_5, m_5^*] = [3]$.

We start with $i = 1$ and $t_1 = 0$. By the first application of Rule A5b, we have $r = 1$ and $f = 0$, and so,

$t_1 = 1$ ([1] counter), $t_2 = 0$ (new counter).

By the second application of Rule A5b ($i = 2$), we have $r = 2$ and $f = 0$, and so,

$t_1 = 2$ ([1] counter), $t_2 = 1$ ([1, 2] counter), $t_3 = 0$ (new counter).

By the third application of Rule A5b ($i = 3$), we have $r = 2$ and $f = 0$, and so,

$t_1 = 3$ ([1] counter), $t_2 = 2$ ([2] counter), $t_3 = 0$ ([1, 2] counter), $t_4 = 0$ (new counter).

By the fourth application of Rule A5b ($i = 4$), we have $r = 4$ and $f = 1$, and so,

$t_1 = 4$ ([1] counter), $t_2 = 3$ ([2] counter), $t_3 = t_4 = 1$ ([1, 2] counters), $t_5 = 0$ (new counter).

By Rule A5a ($i = 5$), we have $m = 3$, $m^* = 1$, $r = 3$ and $s = i\text{-}t_r = 4$ (s is unchanged). The t_1 counter is always a [1] counter (as $[m_1, m_1^*]$ is always [1]), while the t_r counter is the first counter to count $[m, m^*]$ or beyond.

The $\theta(\Omega_\omega$^^4) level separator on pages 12-13 has a sequence of four $[A_{i,p_i}]$ separators, as follows:

$[A_{1,1}] = [1\ [1\ [1\ /_{1,2}\ 2\ /_{2,2}\ 2]\ 2\ /_{2,2}\ 2]\ 2]$, $[m_1, m_1^*] = [1]$,

$[A_{2,1}] = [1\ [1\ /_{1,2}\ 2\ /_{2,2}\ 2]\ 2\ /_{2,2}\ 2]$, $[m_2, m_2^*] = [1, 2]$,

$[A_{3,1}] = [1\ /_{1,2}\ 2\ /_{2,2}\ 2]$, $[m_3, m_3^*] = [1, 2]$,

$[A_{4,1}] = /_{1,2}$, $[m_4, m_4^*] = [1, 2]$.

We start with i = 1 and $t_1 = 0$. By the first application of Rule A5b, we have r = 1 and f = 0, and so,

$t_1 = 1$ ([1] counter), $t_2 = 0$ (new counter).

By the second application of Rule A5b (i = 2), we have r = 2 and f = 0, and so,

$t_1 = 2$ ([1] counter), $t_2 = 1$ ([1, 2] counter), $t_3 = 0$ (new counter).

By the third application of Rule A5b (i = 3), we have r = 3 and f = 1, and so,

$t_1 = 3$ ([1] counter), $t_2 = t_3 = 2$ ([1, 2] counters), $t_4 = 0$ (new counter).

By Rule A5a (i = 4), we have m = 1, m* = 2, r = 2 and s = i-t_r = 2 (s is the same as before).

In a sequence of $[A_{i,p_i}]$ separators meeting the criteria for Rule A5b, $[A_{1,p_1}]$ is always a 1-hyperseparator; $[A_{2,p_2}]$ can be either a 1-, 2- or (1, 2)-hyperseparator; the hyperlevel of $[A_{3,p_3}]$ depends on that of $[A_{2,p_2}]$ – if $[A_{2,p_2}]$ is a 1-hyperseparator, $[A_{3,p_3}]$ can be either a 1-, 2- or (1, 2)-hyperseparator; if $[A_{2,p_2}]$ is a 2-hyperseparator, $[A_{3,p_3}]$ can be either a 1-, 2-, 3- or (1, 2)-hyperseparator; if $[A_{2,p_2}]$ is a (1, 2)-hyperseparator, $[A_{3,p_3}]$ can be either a 1-, 2-, (1, 2)-, (2, 2)- or (1, 3)-hyperseparator. In general, the hyperlevel of $[A_{k,p_k}]$ depends on those of all previous $[A_{i,p_i}]$ separators (i < k) in the sequence; $[A_{k,p_k}]$ can only be an (m, n)-hyperseparator when the previous $[A_{i,p_i}]$ separators have included (j, n)-hyperseparators for all positive integers of 1 ≤ j < m and (1, j)-hyperseparators for all positive integers of 1 ≤ j < n. If $[A_{k,p_k}]$ is an (m, n)-hyperseparator (k ≥ m+n-1, m ≥ 2), then $[A_{k-1,p_{k-1}}]$ is an (m-1, n)- or higher order hyperseparator. If $[A_{k,p_k}]$ is a (1, n)-hyperseparator (k ≥ n, n ≥ 2), then $[A_{k-1,p_{k-1}}]$ is an (1, n-1)- or higher hyperseparator. When going through the sequence of $[A_{i,p_i}]$ separators in reverse order from, say, an (m, n)-hyperseparator, and taking note of those of lower hyperlevels, we always pass through all of the (j, n)-hyperseparators from j = m-1 down to j = 1, then all of the (1, j)-hyperseparators from j = n-1 down to j = 1.

Suppose that after a sequence of $[A_{i,p_i}]$ separators meeting the criteria for Rule A5b, an $[A_{i,p_i}]$ separator meets the criteria for Rule A5a with $[A_{i,p_i}] = /_{1,m^*}$ (m* ≥ 2) for some final value of i ≥ m*, say, i*. If $[A_{i^*-1,p_{i^*-1}}]$ is a (1, m*)- or higher order hyperseparator, there must exist a value j such that $[A_{j,p_j}]$ is a (1, m*)-hyperseparator for some m* ≤ j < i*-1, $[A_{j-1,p_{j-1}}]$ is an (m, m*-1)-hyperseparator (for some positive integer m) and $[A_{k,p_k}]$ is a (1, m*)- or higher hyperseparator for every j ≤ k ≤ i* (by the previous paragraph). Otherwise, $[A_{i^*-1,p_{i^*-1}}]$ is an (m, m*-1)-hyperseparator (for some positive integer m) and we take j = i*. By Rule A5a, t_r = i*-j (number of consecutive (1, m*)- or higher hyperseparators of $[A_{i,p_i}]$ as i increases from j to i*-1 under repeated applications of Rule A5b) and s = i*-t_r = j. This means that $[A_{s,p_s}]$ would be a (1, m*)-hyperseparator and $[A_{s-1,p_{s-1}}]$ an (m, m*-1)-hyperseparator (for some positive integer m). For Rule A5b to have applied with i = s-1,

$[A_{s-1,p_{s-1}}] = [1\ [A_{s,1}]\ 1\ [A_{s,2}]\ ...\ 1\ [A_{s,p_s}]\ c_s\ \#_s]$,

where $p_s \geq 1$ and $c_s \geq 2$. Since neither Rule A5a nor Rule A5a* must have applied,

$[A_{s-1,p_{s-1}}] \neq [1\ /_{1,m^*}\ 2\ _m]$ (as this would 'drop down' to $/_{m,m^*-1}$).

By Rule A5a,

h = EndSub('$\#_s$'),

$\#^*_s$ = DelEndSub('$\#_s$'),

$\#_{n,s}$ = '$\#^*_{s\ h+b-n}$' (1 < n < b),

$\#_{n,k}$ = '$\#_k$' (1 < n ≤ b, s < k ≤ i),

so, taking $\#_s$ = '$\#^*_{s\ h}$', at least one of the following four conditions has to hold true: either $p_s \geq 2$, $c_s \geq 3$, $\#^*_s$ is non-empty or s < i*.

If s = i*, we would have, for n > 1,

$R_{n,s}$ = ‘b ‹$A_{s,1}$’› b [$A_{s,1}$] ... b ‹A_{s,p_s-1}’› b [A_{s,p_s-1}] b ‹$R_{n-1,s}$› b $/_{1,m^*}$ c_s-1 $\#^*_{s\ h+b-n}$’.

$p_s \geq 2$ would mean that [$A_{s,1}$] would be retained within $R_{n,s}$ ([$A_{s,1}$] is a (1, m*)-hyperseparator, since [$A_{s,1}$] ≥ [A_{s,p_s}]), while either $c_s \geq 3$ or $\#^*_s$ being non-empty would result in $/_{1,m^*}$ being kept within $R_{n,s}$.

If s < i*, we would have, for n > 1,

$R_{n,s}$ = ‘b ‹$A_{s,1}$’› b [$A_{s,1}$] ... b ‹A_{s,p_s-1}’› b [A_{s,p_s-1}] b ‹$R_{n,s+1}$› b [A_{s,p_s}] c_s-1 $\#^*_{s\ h+b-n}$’,

$R_{n,k}$ = ‘b ‹$A_{k,1}$’› b [$A_{k,1}$] ... b ‹A_{k,p_k-1}’› b [A_{k,p_k-1}] b ‹$R_{n,k+1}$› b [A_{k,p_k}] c_k-1 $\#_k$’ (s < k < i*),

R_{n,i^*} = ‘b ‹$A_{i^*,1}$’› b [$A_{i^*,1}$] ... b ‹$A_{i^*,p_{i^*}-1}$’› b [$A_{i^*,p_{i^*}-1}$] b ‹$R_{n-1,s}$› b $/_{1,m^*}$ c_{i^*}-1 $\#_{i^*}$’.

Any one of $p_s \geq 2$, $c_s \geq 3$ or $\#^*_s$ being non-empty would lead to the retention of either [$A_{s,1}$] or [A_{s,p_s}] within $R_{n,s}$. But even if $p_s = 1$, $c_s = 2$ and $\#^*_s$ is empty, the fact that [A_{s,p_s}] is a (1, m*)-hyperseparator and [$A_{k,1}$] ≥ [A_{k,p_k}] (for all s < k < i* and $p_k \geq 2$) would still mean than the equivalent of at least one (1, m*)-hyperseparator would be left within $R_{n,s}$ via higher order hyperseparators within the $R_{n,k}$ strings for s < k ≤ i*, unless

[$A_{k,1}$] = [1 [$A_{k+1,1}$] 2] (for all s ≤ k < i*).

But this is impossible, since [$A_{s+1,1}$] is either a (2, m*)- or (1, m*+1)-hyperseparator; [$A_{s+2,1}$] is either a (3, m*)-, (2, m*+1)- or (1, m*+2)-hyperseparator; etc.; and [$A_{i^*,1}$] = $/_{1,m^*}$ (a (1, m*)-hyperseparator).

It follows that a minimum of one (1, m*)-hyperseparator would always be contained within the $R_{n,s}$ string building function in any case. This means that Rule A5a with [A_{i,p_i}] = $/_{1,m^*}$ (m* ≥ 2) would always entail nesting through a series of (h+b-n, m*-1)-hyperseparator angle brackets whenever the $R_{n,s}$ string is built, where h is the subscript at the end of the $\#_s$ string (1 by default). This starts with the $R_{b,s}$ string nesting through (h, m*-1)-hyperseparator brackets and finishes with the $R_{2,s}$ string nesting through (h+b-2, m*-1)-hyperseparator brackets (as $R_{1,s}$ = ‘0’). This system, in fact, optimises the diagonalisation of the notation and, hence, the growth rate of the numbers produced using the notation.

Note that Rule A5a* with [A_{i,p_i}] = [2 $/_{1,k}$ 2 $_m$] (the string $\#_H$ contains the lowest (1, k)-hyperseparator), would mean that R_1 = ‘b [1 $/_{1,k}$ 2 $_{m+b-1}$] b’ = ‘b $/_{m+b-1,k-1}$ b’. By setting m = 1, we achieve R_1 = ‘b $/_{b,k-1}$ b’, which is how the $/_{1,k}$ symbol is reduced to the $/_{b,k-1}$ symbol.

Rule A3 is changed as follows:-

Rule A3 (last entry in any 1-space or higher dimensional space of array is 1):

‘a ‹# [A] 1 $_{m,n}$› b’ = ‘a ‹# $_{m,n}$› b’.

When [A] is an (m_1, m_2)-hyperseparator, [B] is an (n_1, n_2)-hyperseparator and either $m_2 < n_2$; $m_2 = n_2$ and $m_1 < n_1$; or $m_2 = n_2$, $m_1 = n_1$ and level of [A] is less than level of [B],

‘a ‹# [A] 1 [B] #*› b’ = ‘a ‹# [B] #*› b’.

Remove trailing 1’s.

I can now expand the double subscript in the slash symbol so that it too becomes an array in its own right. The single slash with no subscripts has an infinite number of pathways, since

/ = [1 $/_2$ 2] = [1 $/_{1,2}$ 2] = [1 $/_{1,1,2}$ 2] = [1 $/_{1,1,1,2}$ 2] = ...

When there are k subscripts, the (n_1, n_2, ... , n_k)-hyperseparator slash symbol

$/_{n_1,n_2,...,n_k}$ = [1 $/_{n_1+1,n_2,n_3,...,n_k}$ 2]
= [1 $/_{1,n_2+1,n_3,...,n_k}$ 2 $_{n_1}$]
= [1 $/_{1,1,n_3+1,n_4,...,n_k}$ 2 $_{n_1,n_2}$]
= ......
= [1 $/_{1,1,...,1,n_k+1}$ 2 $_{n_1,n_2,...,n_{k-1}}$] (with k-1 1’s)

$= [1\ /_{1,1,...,1,2}\ 2\ _{n_1,n_2,...,n_k}]$ (with k or more 1's)
requires a minimum of $n_1+n_2+...+n_k-(k-1)$ pairs of square brackets around it in order to turn in into a normal separator and be used in the 'base layer' of a curly bracket array. This is known as the Taxicab Rule, in that a separator with a particular hyperlevel requires a greater number of pairs of square brackets than the taxicab (or city-block or Manhattan) distance of the hyperlevel, which is determined by summing the entries (when reduced by 1) of the hyperlevel (when expressed as an array). All 1-hyperseparators have a hyperlevel taxicab distance of zero.

The slash itself in the above equalities (other than the last one) may be substituted by a separator array that contains at least one $(1, 1, ... , 1, r_1, r_2, ...)$-hyperseparator (with at least k 1's and $r_1 \geq 2$) in its 'base layer', for example,

$[X\ /_{1,1,...,1,m}\ Y\ _{n_1,n_2,...,n_k}] = [1\ [X\ /_{1,1,...,1,m}\ Y\ _{1,1,n_3+1,n_4,...,n_k}]\ 2\ _{n_1,n_2}]$
(with k 1's prior to $m \geq 2$; X and Y are strings either side of $/_{1,1,...,1,m}$).

The $\theta(\Omega_{\omega\text{^}2})$ level separator

$\{a, b\ [1\ [2\ /_{1,1,2}\ 2]\ 2]\ 2\} = \{a \langle 0\ [2\ /_{1,1,2}\ 2]\ 2\rangle\ b\}$
$= \{a \langle b \langle b \langle\ ...\ \langle b \langle b\ /_{1,b}\ b\rangle\ b\rangle\ ...\ \rangle\ b\rangle\ b\rangle\ b\}$
(with b pairs of angle brackets).

This is equivalent to

$\{a, b\ \ [1\ [1\ [\ ...\ [1\ [1\ /_{1,b}\ 1, 2]\ 2]\ ...\ ]\ 2]\ 2]\ \ 2\}$ (with b pairs of square brackets).

In general, with k 1's in the subscript,

$\{a, b\ [1\ [c\ /_{1,1,...,1,2}\ 2]\ 2]\ 2\} = \{a \langle 0\ [c\ /_{1,1,...,1,2}\ 2]\ 2\rangle\ b\}$
$= \{a \langle b \langle b \langle\ ...\ \langle b \langle b\ [c\text{-}1\ /_{1,1,...,1,2}\ 2\ _{1,...,1,b}]\ b\rangle\ b\rangle\ ...\ \rangle\ b\rangle\ b\rangle\ b\}$
(with b pairs of angle brackets and k-1 1's in $_{1,...,1,b}$).

When c = 2, the separator $[1\ /_{1,1,...,1,2}\ 2\ _{1,...,1,b}]$ 'drops down' to $/_{1,...,1,b}$ (with k-1 1's in $_{1,...,1,b}$).

Separators with treble subscript slashes begin as follows:

$[1\ \ [2\ /_{1,1,2}\ 2]\ \ 2]$ has level $\theta(\Omega_{\omega\text{^}2})$,
$[1\ \ [2\ /_{1,1,2}\ 2]\ \ 3]$ has level $\theta(\theta_1(\Omega_{\omega\text{^}2}), 1)$,
$[1\ \ [2\ /_{1,1,2}\ 2]\ \ 1\ /\ 2]$ has level $\theta(\theta_1(\Omega_{\omega\text{^}2})+1)$,
$[1\ \ [2\ /_{1,1,2}\ 2]\ \ 1\ \ [2\ /_{1,1,2}\ 2]\ \ 2]$ has level $\theta(\theta_1(\Omega_{\omega\text{^}2})2)$,
$[1\ [2\ \ [2\ /_{1,1,2}\ 2\ _2]\ \ 2]\ 2]$ has level $\theta(\theta_1(\Omega_{\omega\text{^}2})\omega)$,
$[1\ [1\ [2\ /_{1,1,2}\ 2]\ 2\ \ [2\ /_{1,1,2}\ 2\ _2]\ \ 2]\ 2]$ has level $\theta(\theta_1(\Omega_{\omega\text{^}2})\text{^}2)$,
$[1\ [1\ /_2\ 2\ \ [2\ /_{1,1,2}\ 2\ _2]\ \ 2]\ 2]$ has level $\theta(\varepsilon(\theta_1(\Omega_{\omega\text{^}2})+1))$,
$[1\ [1\ [2\ /_{1,2}\ 2\ _2]\ 2\ \ [2\ /_{1,1,2}\ 2\ _2]\ \ 2]\ 2]$ has level $\theta(\theta_1(\theta_2(\Omega_\omega), \theta_1(\Omega_{\omega\text{^}2})+1))$,
$[1\ [1\ [1\ [2\ /_{1,3}\ 2]\ 2\ _2]\ 2\ \ [2\ /_{1,1,2}\ 2\ _2]\ \ 2]\ 2]$ has level $\theta(\theta_1(\theta_2(\Omega_{\omega 2}), \theta_1(\Omega_{\omega\text{^}2})+1))$,
$[1\ [1\ [1\ [1\ [2\ /_{1,4}\ 2]\ 2]\ 2\ _2]\ 2\ \ [2\ /_{1,1,2}\ 2\ _2]\ \ 2]\ 2]$ has level $\theta(\theta_1(\theta_2(\Omega_{\omega 3}), \theta_1(\Omega_{\omega\text{^}2})+1))$,
$[1\ [1\ \ [2\ /_{1,1,2}\ 2\ _2]\ \ 3]\ 2]$ has level $\theta(\theta_2(\Omega_{\omega\text{^}2}), 1)$,
$[1\ [1\ [1\ \ [2\ /_{1,1,2}\ 2\ _3]\ \ 3]\ 2]\ 2]$ has level $\theta(\theta_3(\Omega_{\omega\text{^}2}), 1)$,
$[1\ [1\ [1\ [1\ \ [2\ /_{1,1,2}\ 2\ _4]\ \ 3]\ 2]\ 2]\ 2]$ has level $\theta(\theta_4(\Omega_{\omega\text{^}2}), 1)$,
$[1\ [2\ \ [2\ /_{1,1,2}\ 2\ _{1,2}]\ \ 2]\ 2]$ has level $\theta(\theta_\omega(\Omega_{\omega\text{^}2}), 1)$ ($[2\ /_{1,1,2}\ 2] = [1\ [2\ /_{1,1,2}\ 2\ _{1,2}]\ 2]$),
$[1\ [1\ /_{1,2}\ 2\ \ [2\ /_{1,1,2}\ 2\ _{1,2}]\ \ 2]\ 2]$ has level $\theta(\theta_\omega(\Omega_{\omega\text{^}2})+1)$,
$[1\ [1\ [2\ /_{1,3}\ 2]\ 2\ \ [2\ /_{1,1,2}\ 2\ _{1,2}]\ \ 2]\ 2]$ has level $\theta(\theta_\omega(\Omega_{\omega\text{^}2})+\theta_\omega(\Omega_{\omega 2}))$,
$[1\ [1\ [1\ [2\ /_{1,4}\ 2]\ 2]\ 2\ \ [2\ /_{1,1,2}\ 2\ _{1,2}]\ \ 2]\ 2]$ has level $\theta(\theta_\omega(\Omega_{\omega\text{^}2})+\theta_\omega(\Omega_{\omega 3}))$,
$[1\ [1\ \ [2\ /_{1,1,2}\ 2\ _{1,2}]\ \ 3]\ 2]$ has level $\theta(\theta_\omega(\Omega_{\omega\text{^}2})2)$,
$[1\ [1\ \ [2\ /_{1,1,2}\ 2\ _{1,2}]\ \ 1\ \ [2\ /_{1,1,2}\ 2\ _{1,2}]\ \ 2]\ 2]$ has level $\theta(\theta_\omega(\Omega_{\omega\text{^}2})\text{^}2)$,
$[1\ [1\ [2\ \ [2\ /_{1,1,2}\ 2\ _{2,2}]\ \ 2]\ 2]\ 2]$ has level $\theta(\theta_\omega(\Omega_{\omega\text{^}2})\text{^}\omega)$,
$[1\ [1\ [1\ /_{1,2}\ 2\ \ [2\ /_{1,1,2}\ 2\ _{2,2}]\ \ 2]\ 2]\ 2]$ has level $\theta(\theta_\omega(\Omega_{\omega\text{^}2})\text{^}\Omega_\omega)$,

[1 [1 [1 [2 $/_{1,1,2}$ 2 $_{1,2}$] 2 [2 $/_{1,1,2}$ 2 $_{2,2}$] 2] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega\hat{}2})\hat{}\theta_\omega(\Omega_{\omega\hat{}2}))$,
[1 [1 [1 $/_{2,2}$ 2 [2 $/_{1,1,2}$ 2 $_{2,2}$] 2] 2] 2] has level $\theta(\varepsilon(\theta_\omega(\Omega_{\omega\hat{}2})+1))$ ($/_{2,2}$ = [1 $/_{1,3}$ 2 $_2$]),
[1 [1 [1 [2 $/_{1,3}$ 2 $_2$] 2 [2 $/_{1,1,2}$ 2 $_{2,2}$] 2] 2] 2] has level $\theta(\theta_\omega(\theta_{\omega+1}(\Omega_{\omega2}), \theta_\omega(\Omega_{\omega\hat{}2})+1))$,
[1 [1 [1 [1 [2 $/_{1,4}$ 2] 2 $_2$] 2 [2 $/_{1,1,2}$ 2 $_{2,2}$] 2] 2] 2] has level $\theta(\theta_\omega(\theta_{\omega+1}(\Omega_{\omega3}), \theta_\omega(\Omega_{\omega\hat{}2})+1))$,
[1 [1 [1 [2 $/_{1,1,2}$ 2 $_{2,2}$] 3] 2] 2] has level $\theta(\theta_{\omega+1}(\Omega_{\omega\hat{}2}), 1)$,
[1 [1 [1 [1 [2 $/_{1,1,2}$ 2 $_{3,2}$] 3] 2] 2] 2] has level $\theta(\theta_{\omega+2}(\Omega_{\omega\hat{}2}), 1)$,
[1 [1 [1 [1 [1 [2 $/_{1,1,2}$ 2 $_{4,2}$] 3] 2] 2] 2] 2] has level $\theta(\theta_{\omega+3}(\Omega_{\omega\hat{}2}), 1)$,
[1 [1 [2 [2 $/_{1,1,2}$ 2 $_{1,3}$] 2] 2] 2] has level $\theta(\theta_{\omega2}(\Omega_{\omega\hat{}2}), 1)$ ([2 $/_{1,1,2}$ 2 $_{1,n}$] = [1 [2 $/_{1,1,2}$ 2 $_{1,n+1}$] 2]),
[1 [1 [1 [2 $/_{1,1,2}$ 2 $_{1,3}$] 3] 2] 2] has level $\theta(\theta_{\omega2}(\Omega_{\omega\hat{}2})2)$,
[1 [1 [1 [2 $/_{1,1,2}$ 2 $_{1,3}$] 1 [2 $/_{1,1,2}$ 2 $_{1,3}$] 2] 2] 2] has level $\theta(\theta_{\omega2}(\Omega_{\omega\hat{}2})\hat{}2)$,
[1 [1 [1 [1 $/_{2,3}$ 2 [2 $/_{1,1,2}$ 2 $_{2,3}$] 2] 2] 2] 2] has level $\theta(\varepsilon(\theta_{\omega2}(\Omega_{\omega\hat{}2})+1))$,
[1 [1 [1 [1 [2 $/_{1,1,2}$ 2 $_{2,3}$] 3] 2] 2] 2] has level $\theta(\theta_{\omega2+1}(\Omega_{\omega\hat{}2}), 1)$,
[1 [1 [1 [1 [1 [2 $/_{1,1,2}$ 2 $_{3,3}$] 3] 2] 2] 2] 2] has level $\theta(\theta_{\omega2+2}(\Omega_{\omega\hat{}2}), 1)$,
[1 [1 [1 [1 [1 [1 [2 $/_{1,1,2}$ 2 $_{4,3}$] 3] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega2+3}(\Omega_{\omega\hat{}2}), 1)$,
[1 [1 [1 [2 [2 $/_{1,1,2}$ 2 $_{1,4}$] 2] 2] 2] 2] has level $\theta(\theta_{\omega3}(\Omega_{\omega\hat{}2}), 1)$,
[1 [1 [1 [1 [2 [2 $/_{1,1,2}$ 2 $_{1,5}$] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega4}(\Omega_{\omega\hat{}2}), 1)$,
[1 [1 [1 [1 [1 [2 [2 $/_{1,1,2}$ 2 $_{1,6}$] 2] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega5}(\Omega_{\omega\hat{}2}), 1)$.

Beyond the [2 $/_{1,1,2}$ 2 $_{m,n}$] family, I achieve
[1 [3 $/_{1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega\hat{}2}, 1)$,
[1 [1 / 2 $/_{1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega\hat{}2}, \Omega)$,
[1 [1 $/_2$ 2 $/_{1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega\hat{}2}, \Omega_2)$,
[1 [1 [1 $/_3$ 2 $/_{1,1,2}$ 2 $_2$] 2 $/_{1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega\hat{}2}, \Omega_3)$,
[1 [1 [1 [1 $/_4$ 2 $/_{1,1,2}$ 2 $_3$] 2 $/_{1,1,2}$ 2 $_2$] 2 $/_{1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega\hat{}2}, \Omega_4)$,
[1 [1 $/_{1,2}$ 2 $/_{1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega\hat{}2}, \Omega_\omega)$,
[1 [1 [1 $/_{2,2}$ 2 $/_{1,1,2}$ 2 $_{1,2}$] 2 $/_{1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega\hat{}2}, \Omega_{\omega+1})$,
[1 [1 [1 [1 $/_{3,2}$ 2 $/_{1,1,2}$ 2 $_{2,2}$] 2 $/_{1,1,2}$ 2 $_{1,2}$] 2 $/_{1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega\hat{}2}, \Omega_{\omega+2})$,
[1 [1 [1 $/_{1,3}$ 2 $/_{1,1,2}$ 2 $_{1,2}$] 2 $/_{1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega\hat{}2}, \Omega_{\omega2})$,
[1 [1 [1 [1 $/_{2,3}$ 2 $/_{1,1,2}$ 2 $_{1,3}$] 2 $/_{1,1,2}$ 2 $_{1,2}$] 2 $/_{1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega\hat{}2}, \Omega_{\omega2+1})$,
[1 [1 [1 [1 [1 $/_{3,3}$ 2 $/_{1,1,2}$ 2 $_{2,3}$] 2 $/_{1,1,2}$ 2 $_{1,3}$] 2 $/_{1,1,2}$ 2 $_{1,2}$] 2 $/_{1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega\hat{}2}, \Omega_{\omega2+2})$,
[1 [1 [1 [1 $/_{1,4}$ 2 $/_{1,1,2}$ 2 $_{1,3}$] 2 $/_{1,1,2}$ 2 $_{1,2}$] 2 $/_{1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega\hat{}2}, \Omega_{\omega3})$,
[1 [1 [1 [1 [1 $/_{1,5}$ 2 $/_{1,1,2}$ 2 $_{1,4}$] 2 $/_{1,1,2}$ 2 $_{1,3}$] 2 $/_{1,1,2}$ 2 $_{1,2}$] 2 $/_{1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega\hat{}2}, \Omega_{\omega4})$.

[1 [1 $/_{1,1,2}$ 3] 2] has level $\theta(\Omega_{\omega\hat{}2}+1)$,
[1 [1 $/_{1,1,2}$ 3] 3] has level $\theta(\theta_1(\Omega_{\omega\hat{}2}+1), 1)$,
[1 [1 $/_{1,1,2}$ 3] 1 [1 $/_{1,1,2}$ 3] 2] has level $\theta(\theta_1(\Omega_{\omega\hat{}2}+1)2)$,
[1 [1 [1 $/_{1,1,2}$ 3] 2 [1 $/_{1,1,2}$ 3 $_2$] 2] 2] has level $\theta(\theta_1(\Omega_{\omega\hat{}2}+1)\hat{}2)$,
and, taking $\alpha = \theta_1(\Omega_{\omega\hat{}2}+1)+1$,
[1 [1 $/_2$ 2 [1 $/_{1,1,2}$ 3 $_2$] 2] 2] has level $\theta(\varepsilon_\alpha)$,
[1 [1 [2 $/_{1,1,2}$ 2 $_2$] 2 [1 $/_{1,1,2}$ 3 $_2$] 2] 2] has level $\theta(\theta_1(\theta_2(\Omega_{\omega\hat{}2}), \alpha))$,
[1 [1 [1 $/_n$ 2 $/_{1,1,2}$ 2 $_2$] 2 [1 $/_{1,1,2}$ 3 $_2$] 2] 2] ($1 \le n \le 3$) has level $\theta(\theta_1(\theta_2(\Omega_{\omega\hat{}2}, \Omega_n), \alpha))$,
[1 [1 [1 [1 $/_4$ 2 $/_{1,1,2}$ 2 $_3$] 2 $/_{1,1,2}$ 2 $_2$] 2 [1 $/_{1,1,2}$ 3 $_2$] 2] 2] has level $\theta(\theta_1(\theta_2(\Omega_{\omega\hat{}2}, \Omega_4), \alpha))$,
[1 [1 [1 $/_{1,2}$ 2 $/_{1,1,2}$ 2 $_2$] 2 [1 $/_{1,1,2}$ 3 $_2$] 2] 2] has level $\theta(\theta_1(\theta_2(\Omega_{\omega\hat{}2}, \Omega_\omega), \alpha))$,
[1 [1 [1 $/_{2,2}$ 2 $/_{1,1,2}$ 2 $_2$] 2 [1 $/_{1,1,2}$ 3 $_2$] 2] 2] has level $\theta(\theta_1(\theta_2(\Omega_{\omega\hat{}2}, \Omega_{\omega+1}), \alpha))$,
[1 [1 [1 [1 $/_{3,2}$ 2 $/_{1,1,2}$ 2 $_{2,2}$] 2 $/_{1,1,2}$ 2 $_2$] 2 [1 $/_{1,1,2}$ 3 $_2$] 2] 2] has level $\theta(\theta_1(\theta_2(\Omega_{\omega\hat{}2}, \Omega_{\omega+2}), \alpha))$,
[1 [1 [1 [1 $/_{1,3}$ 2 $/_{1,1,2}$ 2 $_{2,2}$] 2 $/_{1,1,2}$ 2 $_2$] 2 [1 $/_{1,1,2}$ 3 $_2$] 2] 2] has level $\theta(\theta_1(\theta_2(\Omega_{\omega\hat{}2}, \Omega_{\omega2}), \alpha))$,
[1 [1 [1 [1 $/_{2,3}$ 2 $/_{1,1,2}$ 2 $_{2,2}$] 2 $/_{1,1,2}$ 2 $_2$] 2 [1 $/_{1,1,2}$ 3 $_2$] 2] 2] has level $\theta(\theta_1(\theta_2(\Omega_{\omega\hat{}2}, \Omega_{\omega2+1}), \alpha))$,
[1 [1 [1 [1 [1 $/_{2,4}$ 2 $/_{1,1,2}$ 2 $_{2,3}$] 2 $/_{1,1,2}$ 2 $_{2,2}$] 2 $/_{1,1,2}$ 2 $_2$] 2 [1 $/_{1,1,2}$ 3 $_2$] 2] 2] has level
$\theta(\theta_1(\theta_2(\Omega_{\omega\hat{}2}, \Omega_{\omega3+1}), \alpha))$ (nesting through [$_{2,n}$] brackets),

[1 [1 [1 $/_{1,1,2}$ 3 $_{2}$] 3] 2] has level $\theta(\theta_2(\Omega_{\omega^2}+1), 1)$,
[1 [1 [1 [1 $/_{1,1,2}$ 3 $_{3}$] 3] 2] 2] has level $\theta(\theta_3(\Omega_{\omega^2}+1), 1)$,
[1 [1 [1 $/_{1,1,2}$ 3 $_{1,2}$] 3] 2] has level $\theta(\theta_\omega(\Omega_{\omega^2}+1), 1)$,
[1 [1 [1 [1 $/_{1,1,2}$ 3 $_{1,3}$] 3] 2] 2] has level $\theta(\theta_{\omega2}(\Omega_{\omega^2}+1), 1)$,
[1 [2 $/_{1,1,2}$ 3] 2] has level $\theta(\Omega_{\omega^2}+1, 1)$,
[1 [1 $/_{1,1,2}$ 4] 2] has level $\theta(\Omega_{\omega^2}+2)$,
[1 [1 $/_{1,1,2}$ 1 $/_{1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^2}2)$,
[1 [1 $/_{1,1,2}$ 1 $/_{1,1,2}$ 1 $/_{1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^2}{}^{2})$,
[1 [1 [2 $/_{2,1,2}$ 2] 2] 2] has level $\theta(\Omega_{\omega^2}{}^{\omega})$,
[1 [1 [1 $/_{1,1,2}$ 2 $/_{2,1,2}$ 2] 2] 2] has level $\theta(\Omega_{\omega^2}{}^{\Omega_{\omega^2}})$,
[1 [1 [1 $/_{2,1,2}$ 3] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega^2}+1)) = \theta(\theta_{\omega^2}(1))$,
[1 [1 [1 $/_{2,1,2}$ 1 $/_{2,1,2}$ 2] 2] 2] has level $\theta(\zeta(\Omega_{\omega^2}+1)) = \theta(\theta_{\omega^2}(2))$,
[1 [1 [1 [2 $/_{3,1,2}$ 2] 2] 2] 2] has level $\theta(\theta_{\omega^2}(\omega))$,
[1 [1 [1 [1 / 2 $/_{3,1,2}$ 2] 2] 2] 2] has level $\theta(\theta_{\omega^2}(\Omega))$,
[1 [1 [1 [1 $/_{1,1,2}$ 2 $/_{3,1,2}$ 2] 2] 2] 2] has level $\theta(\theta_{\omega^2}(\Omega_{\omega^2}))$,
[1 [1 [1 [1 $/_{2,1,2}$ 2 $/_{3,1,2}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega^2+1})$,
[1 [1 [1 [1 $/_{3,1,2}$ 3] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega^2+1}+1)) = \theta(\theta_{\omega^2+1}(1))$,
[1 [1 [1 [1 $/_{3,1,2}$ 1 $/_{3,1,2}$ 2] 2] 2] 2] has level $\theta(\zeta(\Omega_{\omega^2+1}+1)) = \theta(\theta_{\omega^2+1}(2))$,
[1 [1 [1 [1 [2 $/_{4,1,2}$ 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega^2+1}(\omega))$,
[1 [1 [1 [1 [1 $/_{3,1,2}$ 2 $/_{4,1,2}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega^2+2})$,
[1 [1 [1 [1 [1 [1 $/_{4,1,2}$ 2 $/_{5,1,2}$ 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega^2+3})$.

[1 [1 [2 $/_{1,2,2}$ 2] 2] 2] has level $\theta(\Omega_{\omega^2+\omega})$ ($/_{1,n,2}$ = [1 $/_{1,n+1,2}$ 2]),
[1 [1 [3 $/_{1,2,2}$ 2] 2] 2] has level $\theta(\Omega_{\omega^2+\omega}, 1)$,
[1 [1 [1 $/_{1,1,2}$ 2 $/_{1,2,2}$ 2] 2] 2] has level $\theta(\Omega_{\omega^2+\omega}, \Omega_{\omega^2})$,
[1 [1 [1 $/_{2,1,2}$ 2 $/_{1,2,2}$ 2] 2] 2] has level $\theta(\Omega_{\omega^2+\omega}, \Omega_{\omega^2+1})$,
[1 [1 [1 [1 $/_{3,1,2}$ 2 $/_{1,2,2}$ 2 $_{2}$] 2 $/_{1,2,2}$ 2] 2] 2] has level $\theta(\Omega_{\omega^2+\omega}, \Omega_{\omega^2+2})$,
[1 [1 [1 [1 [1 $/_{4,1,2}$ 2 $/_{1,2,2}$ 2 $_{3}$] 2 $/_{1,2,2}$ 2 $_{2}$] 2 $/_{1,2,2}$ 2] 2] 2] has level $\theta(\Omega_{\omega^2+\omega}, \Omega_{\omega^2+3})$,
[1 [1 [1 $/_{1,2,2}$ 3] 2] 2] has level $\theta(\Omega_{\omega^2+\omega}+1)$,
[1 [1 [1 $/_{1,2,2}$ 1 $/_{1,2,2}$ 2] 2] 2] has level $\theta(\Omega_{\omega^2+\omega}2)$,
[1 [1 [1 $/_{1,2,2}$ 1 $/_{1,2,2}$ 1 $/_{1,2,2}$ 2] 2] 2] has level $\theta(\Omega_{\omega^2+\omega}{}^{2})$,
[1 [1 [1 [2 $/_{2,2,2}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega^2+\omega}{}^{\omega})$,
[1 [1 [1 [1 $/_{1,2,2}$ 2 $/_{2,2,2}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega^2+\omega}{}^{\Omega_{\omega^2+\omega}})$,
[1 [1 [1 [1 $/_{2,2,2}$ 3] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega^2+\omega}+1)) = \theta(\theta_{\omega^2+\omega}(1))$,
[1 [1 [1 [1 $/_{2,2,2}$ 1 $/_{2,2,2}$ 2] 2] 2] 2] has level $\theta(\zeta(\Omega_{\omega^2+\omega}+1)) = \theta(\theta_{\omega^2+\omega}(2))$,
[1 [1 [1 [1 [2 $/_{3,2,2}$ 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega^2+\omega}(\omega))$,
[1 [1 [1 [1 [1 $/_{2,2,2}$ 2 $/_{3,2,2}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega^2+\omega+1})$,
[1 [1 [1 [1 [1 [1 $/_{3,2,2}$ 2 $/_{4,2,2}$ 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega^2+\omega+2})$,
[1 [1 [1 [1 [1 [1 [1 $/_{4,2,2}$ 2 $/_{5,2,2}$ 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega^2+\omega+3})$,
[1 [1 [1 [2 $/_{1,3,2}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega^2+\omega2})$,
[1 [1 [1 [1 [2 $/_{1,4,2}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega^2+\omega3})$,
[1 [1 [1 [1 [1 [2 $/_{1,5,2}$ 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega^2+\omega4})$.

[1 [1 [2 $/_{1,1,3}$ 2] 2] 2] has level $\theta(\Omega_{(\omega^2)2})$ ($/_{1,1,n}$ = [1 $/_{1,1,n+1}$ 2]),
[1 [1 [3 $/_{1,1,3}$ 2] 2] 2] has level $\theta(\Omega_{(\omega^2)2}, 1)$,
[1 [1 [1 $/_{1,1,2}$ 2 $/_{1,1,3}$ 2] 2] 2] has level $\theta(\Omega_{(\omega^2)2}, \Omega_{\omega^2})$,
[1 [1 [1 $/_{2,1,2}$ 2 $/_{1,1,3}$ 2] 2] 2] has level $\theta(\Omega_{(\omega^2)2}, \Omega_{\omega^2+1})$,
[1 [1 [1 [1 $/_{3,1,2}$ 2 $/_{1,1,3}$ 2 $_{2}$] 2 $/_{1,1,3}$ 2] 2] 2] has level $\theta(\Omega_{(\omega^2)2}, \Omega_{\omega^2+2})$,
[1 [1 [1 [1 [1 $/_{4,1,2}$ 2 $/_{1,1,3}$ 2 $_{3}$] 2 $/_{1,1,3}$ 2 $_{2}$] 2 $/_{1,1,3}$ 2] 2] 2] has level $\theta(\Omega_{(\omega^2)2}, \Omega_{\omega^2+3})$,

[1 [1 [1 $/_{1,2,2}$ 2 $/_{1,1,3}$ 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2}, \Omega_{\omega\wedge 2+\omega})$,
[1 [1 [1 [1 $/_{2,2,2}$ 2 $/_{1,1,3}$ 2 $_{1,2}$] 2 $/_{1,1,3}$ 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2}, \Omega_{\omega\wedge 2+\omega+1})$,
[1 [1 [1 [1 [1 $/_{3,2,2}$ 2 $/_{1,1,3}$ 2 $_{2,2}$] 2 $/_{1,1,3}$ 2 $_{1,2}$] 2 $/_{1,1,3}$ 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2}, \Omega_{\omega\wedge 2+\omega+2})$,
[1 [1 [1 [1 $/_{1,3,2}$ 2 $/_{1,1,3}$ 2 $_{1,2}$] 2 $/_{1,1,3}$ 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2}, \Omega_{\omega\wedge 2+\omega 2})$,
[1 [1 [1 [1 [1 $/_{1,4,2}$ 2 $/_{1,1,3}$ 2 $_{1,3}$] 2 $/_{1,1,3}$ 2 $_{1,2}$] 2 $/_{1,1,3}$ 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2}, \Omega_{\omega\wedge 2+\omega 3})$,
[1 [1 [1 $/_{1,1,3}$ 3] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2}+1)$,
[1 [1 [1 $/_{1,1,3}$ 1 $/_{1,1,3}$ 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2}2)$,
[1 [1 [1 [2 $/_{2,1,3}$ 2] 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2}\wedge\omega)$,
[1 [1 [1 [1 $/_{1,1,3}$ 2 $/_{2,1,3}$ 2] 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2}\wedge\Omega_{(\omega\wedge 2)2})$,
[1 [1 [1 [1 $/_{2,1,3}$ 3] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{(\omega\wedge 2)2}+1)) = \theta(\theta_{(\omega\wedge 2)2}(1))$,
[1 [1 [1 [1 $/_{2,1,3}$ 1 $/_{2,1,3}$ 2] 2] 2] 2] has level $\theta(\zeta(\Omega_{(\omega\wedge 2)2}+1)) = \theta(\theta_{(\omega\wedge 2)2}(2))$,
[1 [1 [1 [1 [2 $/_{3,1,3}$ 2] 2] 2] 2] 2] has level $\theta(\theta_{(\omega\wedge 2)2}(\omega))$,
[1 [1 [1 [1 [1 $/_{2,1,3}$ 2 $/_{3,1,3}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2+1})$,
[1 [1 [1 [1 [1 $/_{3,1,3}$ 3] 2] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{(\omega\wedge 2)2+1}+1)) = \theta(\theta_{(\omega\wedge 2)2+1}(1))$,
[1 [1 [1 [1 [1 [1 $/_{3,1,3}$ 2 $/_{4,1,3}$ 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2+2})$,
[1 [1 [1 [1 [1 [1 [1 $/_{4,1,3}$ 2 $/_{5,1,3}$ 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2+3})$,
[1 [1 [1 [2 $/_{1,2,3}$ 2] 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2+\omega})$,
[1 [1 [1 [3 $/_{1,2,3}$ 2] 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2+\omega}, 1)$,
[1 [1 [1 [1 $/_{1,2,3}$ 3] 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2+\omega}+1)$,
[1 [1 [1 [1 $/_{1,2,3}$ 1 $/_{1,2,3}$ 2] 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2+\omega}2)$,
[1 [1 [1 [1 [1 $/_{1,2,3}$ 2 $/_{2,2,3}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2+\omega}\wedge\Omega_{(\omega\wedge 2)2+\omega})$,
[1 [1 [1 [1 [1 $/_{2,2,3}$ 3] 2] 2] 2] 2] has level $\theta(\varepsilon(\Omega_{(\omega\wedge 2)2+\omega}+1)) = \theta(\theta_{(\omega\wedge 2)2+\omega}(1))$,
[1 [1 [1 [1 [1 [1 $/_{2,2,3}$ 2 $/_{3,2,3}$ 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2+\omega+1})$,
[1 [1 [1 [1 [1 [1 [1 $/_{3,2,3}$ 2 $/_{4,2,3}$ 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2+\omega+2})$,
[1 [1 [1 [1 [2 $/_{1,3,3}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2+\omega 2})$,
[1 [1 [1 [1 [1 [2 $/_{1,4,3}$ 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2+\omega 3})$,
[1 [1 [1 [1 [1 [1 [2 $/_{1,5,3}$ 2] 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)2+\omega 4})$,
[1 [1 [1 [2 $/_{1,1,4}$ 2] 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)3})$,
[1 [1 [1 [1 [2 $/_{1,1,5}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)4})$,
[1 [1 [1 [1 [1 [2 $/_{1,1,6}$ 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{(\omega\wedge 2)5})$.

The sequence of separators starting with the last three has limit ordinal $\theta(\Omega_{\omega\wedge 3})$.

Replacing [2 $/_{1,1,2}$ 2 $_{m,n}$] by [2 $/_{1,1,1,2}$ 2 $_{m,n}$] in the list of separators on pages 29-30 entails changing each $\Omega_{\omega\wedge 2}$ in the associated ordinal levels to $\Omega_{\omega\wedge 3}$. Significant examples are as follows:

[1 [2 $/_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega\wedge 3})$,
[1 [2 $/_{1,1,1,2}$ 2] 3] has level $\theta(\theta_1(\Omega_{\omega\wedge 3}), 1)$,
[1 [2 $/_{1,1,1,2}$ 2] 1 / 2] has level $\theta(\theta_1(\Omega_{\omega\wedge 3})+1)$,
[1 [2 $/_{1,1,1,2}$ 2] 1 [2 $/_{1,1,1,2}$ 2] 2] has level $\theta(\theta_1(\Omega_{\omega\wedge 3})2)$,
[1 [1 $/_2$ 2 [2 $/_{1,1,1,2}$ 2 $_2$] 2] 2] has level $\theta(\varepsilon(\theta_1(\Omega_{\omega\wedge 3})+1))$,
[1 [1 [2 $/_{1,1,1,2}$ 2 $_2$] 3] 2] has level $\theta(\theta_2(\Omega_{\omega\wedge 3}), 1)$,
[1 [1 [1 [2 $/_{1,1,1,2}$ 2 $_3$] 3] 2] 2] has level $\theta(\theta_3(\Omega_{\omega\wedge 3}), 1)$,
[1 [2 [2 $/_{1,1,1,2}$ 2 $_{1,2}$] 2] 2] has level $\theta(\theta_\omega(\Omega_{\omega\wedge 3}), 1)$ ([2 $/_{1,1,1,2}$ 2] = [1 [2 $/_{1,1,1,2}$ 2 $_{1,2}$] 2]),
[1 [1 [1 [2 $/_{1,1,1,2}$ 2 $_{2,2}$] 3] 2] 2] has level $\theta(\theta_{\omega+1}(\Omega_{\omega\wedge 3}), 1)$,
[1 [1 [1 [1 [2 $/_{1,1,1,2}$ 2 $_{3,2}$] 3] 2] 2] 2] has level $\theta(\theta_{\omega+2}(\Omega_{\omega\wedge 3}), 1)$,
[1 [1 [2 [2 $/_{1,1,1,2}$ 2 $_{1,3}$] 2] 2] 2] has level $\theta(\theta_{\omega 2}(\Omega_{\omega\wedge 3}), 1)$,
[1 [1 [1 [2 [2 $/_{1,1,1,2}$ 2 $_{1,4}$] 2] 2] 2] 2] has level $\theta(\theta_{\omega 3}(\Omega_{\omega\wedge 3}), 1)$,
[1 [1 [1 [1 [2 [2 $/_{1,1,1,2}$ 2 $_{1,5}$] 2] 2] 2] 2] 2] has level $\theta(\theta_{\omega 4}(\Omega_{\omega\wedge 3}), 1)$.

A treble subscript to the closed bracket of [2 /$_{1,1,1,2}$ 2] leads to

[1 [2 [2 /$_{1,1,1,2}$ 2 $_{1,1,2}$] 2] 2] has level $\theta(\theta_{\omega^2}(\Omega_{\omega^3}), 1)$ ([2 /$_{1,1,1,2}$ 2] = [1 [2 /$_{1,1,1,2}$ 2 $_{1,1,2}$] 2]),
[1 [1 [1 [2 /$_{1,1,1,2}$ 2 $_{2,1,2}$] 3] 2] 2] has level $\theta(\theta_{\omega^2+1}(\Omega_{\omega^3}), 1)$,
[1 [1 [1 [1 [2 /$_{1,1,1,2}$ 2 $_{3,1,2}$] 3] 2] 2] 2] has level $\theta(\theta_{\omega^2+2}(\Omega_{\omega^3}), 1)$,
[1 [1 [2 [2 /$_{1,1,1,2}$ 2 $_{1,2,2}$] 2] 2] 2] has level $\theta(\theta_{\omega^2+\omega}(\Omega_{\omega^3}), 1)$,
[1 [1 [1 [2 [2 /$_{1,1,1,2}$ 2 $_{1,3,2}$] 2] 2] 2] 2] has level $\theta(\theta_{\omega^2+\omega 2}(\Omega_{\omega^3}), 1)$,
[1 [1 [2 [2 /$_{1,1,1,2}$ 2 $_{1,1,3}$] 2] 2] 2] has level $\theta(\theta_{(\omega^2)2}(\Omega_{\omega^3}), 1)$,
[1 [1 [1 [2 [2 /$_{1,1,1,2}$ 2 $_{1,1,4}$] 2] 2] 2] 2] has level $\theta(\theta_{(\omega^2)3}(\Omega_{\omega^3}), 1)$.

Beyond the [2 /$_{1,1,1,2}$ 2 $_{k,m,n}$] family, I achieve

[1 [3 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}, 1)$,
[1 [1 / 2 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}, \Omega)$,
[1 [1 /$_2$ 2 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}, \Omega_2)$,
[1 [1 [1 /$_3$ 2 /$_{1,1,1,2}$ 2 $_2$] 2 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}, \Omega_3)$,
[1 [1 [1 [1 /$_4$ 2 /$_{1,1,1,2}$ 2 $_3$] 2 /$_{1,1,1,2}$ 2 $_2$] 2 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}, \Omega_4)$,
[1 [1 /$_{1,2}$ 2 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}, \Omega_\omega)$,
[1 [1 [1 /$_{2,2}$ 2 /$_{1,1,1,2}$ 2 $_{1,2}$] 2 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}, \Omega_{\omega+1})$,
[1 [1 [1 [1 /$_{3,2}$ 2 /$_{1,1,1,2}$ 2 $_{2,2}$] 2 /$_{1,1,1,2}$ 2 $_{1,2}$] 2 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}, \Omega_{\omega+2})$,
[1 [1 [1 /$_{1,3}$ 2 /$_{1,1,1,2}$ 2 $_{1,2}$] 2 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}, \Omega_{\omega 2})$,
[1 [1 [1 [1 /$_{1,4}$ 2 /$_{1,1,1,2}$ 2 $_{1,3}$] 2 /$_{1,1,1,2}$ 2 $_{1,2}$] 2 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}, \Omega_{\omega 3})$,
[1 [1 [1 [1 [1 /$_{1,5}$ 2 /$_{1,1,1,2}$ 2 $_{1,4}$] 2 /$_{1,1,1,2}$ 2 $_{1,3}$] 2 /$_{1,1,1,2}$ 2 $_{1,2}$] 2 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}, \Omega_{\omega 4})$,
[1 [1 /$_{1,1,2}$ 2 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}, \Omega_{\omega^2})$,
[1 [1 [1 /$_{2,1,2}$ 2 /$_{1,1,1,2}$ 2 $_{1,1,2}$] 2 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}, \Omega_{\omega^2+1})$,
[1 [1 [1 [1 /$_{3,1,2}$ 2 /$_{1,1,1,2}$ 2 $_{2,1,2}$] 2 /$_{1,1,1,2}$ 2 $_{1,1,2}$] 2 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}, \Omega_{\omega^2+2})$,
[1 [1 [1 /$_{1,2,2}$ 2 /$_{1,1,1,2}$ 2 $_{1,1,2}$] 2 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}, \Omega_{\omega^2+\omega})$,
[1 [1 [1 [1 /$_{1,3,2}$ 2 /$_{1,1,1,2}$ 2 $_{1,2,2}$] 2 /$_{1,1,1,2}$ 2 $_{1,1,2}$] 2 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}, \Omega_{\omega^2+\omega 2})$,
[1 [1 [1 [1 [1 /$_{1,4,2}$ 2 /$_{1,1,1,2}$ 2 $_{1,3,2}$] 2 /$_{1,1,1,2}$ 2 $_{1,2,2}$] 2 /$_{1,1,1,2}$ 2 $_{1,1,2}$] 2 /$_{1,1,1,2}$ 2] 2] has level
$\theta(\Omega_{\omega^3}, \Omega_{\omega^2+\omega 3})$,
[1 [1 [1 /$_{1,1,3}$ 2 /$_{1,1,1,2}$ 2 $_{1,1,2}$] 2 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}, \Omega_{(\omega^2)2})$,
[1 [1 [1 [1 /$_{1,1,4}$ 2 /$_{1,1,1,2}$ 2 $_{1,1,3}$] 2 /$_{1,1,1,2}$ 2 $_{1,1,2}$] 2 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}, \Omega_{(\omega^2)3})$,
[1 [1 [1 [1 [1 /$_{1,1,5}$ 2 /$_{1,1,1,2}$ 2 $_{1,1,4}$] 2 /$_{1,1,1,2}$ 2 $_{1,1,3}$] 2 /$_{1,1,1,2}$ 2 $_{1,1,2}$] 2 /$_{1,1,1,2}$ 2] 2] has level
$\theta(\Omega_{\omega^3}, \Omega_{(\omega^2)4})$,
[1 [1 /$_{1,1,1,2}$ 3] 2] has level $\theta(\Omega_{\omega^3}+1)$,
[1 [1 /$_{1,1,1,2}$ 1 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}2)$,
[1 [1 /$_{1,1,1,2}$ 1 /$_{1,1,1,2}$ 1 /$_{1,1,1,2}$ 2] 2] has level $\theta(\Omega_{\omega^3}\hat{}2)$,
[1 [1 [2 /$_{2,1,1,2}$ 2] 2] 2] has level $\theta(\Omega_{\omega^3}\hat{}\omega)$,
[1 [1 [1 /$_{1,1,1,2}$ 2 /$_{2,1,1,2}$ 2] 2] 2] has level $\theta(\Omega_{\omega^3}\hat{}\Omega_{\omega^3})$,
[1 [1 [1 /$_{2,1,1,2}$ 3] 2] 2] has level $\theta(\varepsilon(\Omega_{\omega^3}+1)) = \theta(\theta_{\omega^3}(1))$,
[1 [1 [1 /$_{2,1,1,2}$ 1 /$_{2,1,1,2}$ 2] 2] 2] has level $\theta(\zeta(\Omega_{\omega^3}+1)) = \theta(\theta_{\omega^3}(2))$,
[1 [1 [1 [2 /$_{3,1,1,2}$ 2] 2] 2] 2] has level $\theta(\theta_{\omega^3}(\omega))$,
[1 [1 [1 [1 /$_{2,1,1,2}$ 2 /$_{3,1,1,2}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega^3+1})$,
[1 [1 [1 [1 [1 /$_{3,1,1,2}$ 2 /$_{4,1,1,2}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega^3+2})$,
[1 [1 [1 [1 [1 [1 /$_{4,1,1,2}$ 2 /$_{5,1,1,2}$ 2] 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega^3+3})$,
[1 [1 [2 /$_{1,2,1,2}$ 2] 2] 2] has level $\theta(\Omega_{\omega^3+\omega})$ (/$_{1,n,1,2}$ = [1 /$_{1,n+1,1,2}$ 2]),
[1 [1 [1 [2 /$_{1,3,1,2}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega^3+\omega 2})$,
[1 [1 [1 [1 [2 /$_{1,4,1,2}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega^3+\omega 3})$,
[1 [1 [2 /$_{1,1,2,2}$ 2] 2] 2] has level $\theta(\Omega_{\omega^3+\omega^2})$ (/$_{1,1,n,2}$ = [1 /$_{1,1,n+1,2}$ 2]),
[1 [1 [1 [2 /$_{1,1,3,2}$ 2] 2] 2] 2] has level $\theta(\Omega_{\omega^3+(\omega^2)2})$,
[1 [1 [1 [1 [2 /$_{1,1,4,2}$ 2] 2] 2] 2] 2] has level $\theta(\Omega_{\omega^3+(\omega^2)3})$,

$[1\ [1\ [2\ /_{1,1,1,3}\ 2]\ 2]\ 2]$ has level $\theta(\Omega_{(\omega\char94 3)2})$ $(/_{1,1,1,n} = [1\ /_{1,1,1,n+1}\ 2])$,
$[1\ [1\ [1\ [2\ /_{1,1,1,4}\ 2]\ 2]\ 2]\ 2]$ has level $\theta(\Omega_{(\omega\char94 3)3})$,
$[1\ [1\ [1\ [1\ [2\ /_{1,1,1,5}\ 2]\ 2]\ 2]\ 2]\ 2]$ has level $\theta(\Omega_{(\omega\char94 3)4})$.

The sequence of separators starting with the last three has limit ordinal $\theta(\Omega_{\omega\char94 4})$.

With five or more slash subscripts, I have

$[1\ [2\ /_{1,1,1,1,2}\ 2]\ 2]$ has level $\theta(\Omega_{\omega\char94 4})$,
$[1\ [2\ /_{1,1,1,1,1,2}\ 2]\ 2]$ has level $\theta(\Omega_{\omega\char94 5})$,
$[1\ [2\ /_{1,1,\ldots,1,2}\ 2]\ 2]$ (with n 1's) has level $\theta(\Omega_{\omega\char94 n})$.

The limit ordinal of this slash subscript notation is $\theta(\Omega_{\omega\char94 \omega})$.

In general, with k+1 slash subscripts and $m+n_1+n_2+\ldots+n_k-k$ pairs of square brackets ($k \geq 1$, $n_k \geq 2$),

$[1\ [1\ [\ \ldots\ [1\ [2\ /_{m,n_1,n_2,\ldots,n_k}\ 2]\ 2]\ \ldots\ ]\ 2]\ 2]$
has level $\theta(\Omega_\alpha)$ $(m = 1)$, $\theta(\Omega_\alpha\char94 \omega)$ $(m = 2)$, $\theta(\theta_{\alpha+m-3}(\omega))$ $(m \geq 3)$;

$[1\ [1\ [\ \ldots\ [1\ [1\ /_{m-1,n_1,n_2,\ldots,n_k}\ 2\ /_{m,n_1,n_2,\ldots,n_k}\ 2]\ 2]\ \ldots\ ]\ 2]\ 2]$
has level $\theta(\Omega_\alpha\char94 \Omega_\alpha)$ $(m = 2)$, $\theta(\Omega_{\alpha+m-2})$ $(m \geq 3)$;

$[1\ [1\ [\ \ldots\ [1\ [1\ /_{m,n_1,n_2,\ldots,n_k}\ 3]\ 2]\ \ldots\ ]\ 2]\ 2]$
has level $\theta(\Omega_\alpha+1)$ $(m = 1)$, $\theta(\varepsilon(\Omega_{\alpha+m-2}+1)) = \theta(\theta_{\alpha+m-2}(1))$ $(m \geq 2)$;

$[1\ [1\ [\ \ldots\ [1\ [1\ /_{m,n_1,n_2,\ldots,n_k}\ 1\ /_{m,n_1,n_2,\ldots,n_k}\ 2]\ 2]\ \ldots\ ]\ 2]\ 2]$
has level $\theta(\Omega_\alpha 2)$ $(m = 1)$, $\theta(\zeta(\Omega_{\alpha+m-2}+1)) = \theta(\theta_{\alpha+m-2}(2))$ $(m \geq 2)$;

where $\alpha = (\omega\char94 k)(n_k-1) + \ldots + (\omega\char94 2)(n_2-1) + \omega(n_1-1)$.

The recursive definition of an $(n_1, n_2, \ldots, n_k)$-hyperseparator (for $k \geq 1$, $n_1 \geq 1$, $n_i \geq 1$ $(1 \leq i < k)$ and $n_k \geq 2$ $(k \geq 2)$) is that one of the following four conditions hold:

1. It is the $/_{n_1,n_2,\ldots,n_k}$ symbol.
2. Contains an $(n_1+1, n_2, n_3, \ldots, n_k)$-hyperseparator as highest hyperseparator in its 'base layer'.
3. Subscripted by $n_1,n_2,\ldots,n_i$ and contains a $(1, 1, \ldots, 1, n_{i+1}+1, n_{i+2}, n_{i+3}, \ldots, n_k)$-hyperseparator (with i 1's) as highest hyperseparator in its 'base layer', for some $1 \leq i < k$. (When $i = k-1$, the hyperlevel expression would read $(1, 1, \ldots, 1, n_k+1)$-hyperseparator (with k-1 1's).)
4. Subscripted by $n_1,n_2,\ldots,n_k$ and contains a $(1, 1, \ldots, 1, 2)$-hyperseparator (with at least k 1's) as highest hyperseparator in its 'base layer'.

In order to find the minimum number of square brackets around an $(n_1, n_2, \ldots, n_k)$-hyperseparator necessary to turn it into a normal separator, use the Taxicab Rule – take the sum of the n_i's (when reduced by 1), then add 1. This is $n_1+n_2+\ldots+n_k+1-k$ pairs of square brackets. In the case of a slash with subscript array, take the sum of the subscripts (when reduced by 1), then add 1. A pair of square brackets with k subscripts (the last subscript being at least 2) appended to the inside of the closed bracket is only possible when the lowest layer contains a $(1, 1, \ldots, 1, n_1+1, n_2, n_3, \ldots)$-hyperseparator (with at least k 1's) for some positive integers n_i.

Rule A5b now reads as follows:-

Rule A5b (Rule A5a does not apply, separator $[A_{i,p_i}] = [1\ [A_{i+1,1}]\ 1\ [A_{i+1,2}]\ \ldots\ 1\ [A_{i+1,p_{i+1}}]\ c_{i+1}\ \#_{i+1}]$, which is an $(m_{i,1}, m_{i,2}, \ldots, m_{i,q_i})$-hyperseparator, where $p_{i+1} \geq 1$, $c_{i+1} \geq 2$, $q_i \geq 1$, $m_{i,j} \geq 1$ (for all $1 \leq j \leq q_i$) and $m_{i,q_i} \geq 2$ $(q_i \geq 2)$):

S_i = 'b $\langle A_{i,1}' \rangle$ b $[A_{i,1}]$ b $\langle A_{i,2}' \rangle$ b $[A_{i,2}]$ ... b $\langle A_{i,p_i-1}' \rangle$ b $[A_{i,p_i-1}]$ b $\langle S_{i+1} \rangle$ b $[A_{i,p_i}]$ c_i-1 $\#_i$'.

Set r to 1 and f to 0. For each j from 1 to i-1, increment r by 1 when either $q_j < q_i$ or $q_j = q_i$ and there exists a number n such that $m_{j,n} < m_{i,n}$ and $m_{j,k} = m_{i,k}$ (for all $n < k \leq q_i$); and increment r by 1 and set f to 1 when $q_j = q_i$ and $m_{j,k} = m_{i,k}$ (for all $1 \leq k \leq q_i$).

Increment t_1, t_2, ... , t_r by 1; set t_r to t_{r-1} if f = 1; reset t_{r+1}, t_{r+2}, ... , t_i to 0; set t_{i+1} to 0; increment i by 1 (so that i = t_1+1) and repeat Rules A5a-c.

In Rule A5b, $[A_{1,p_1}]$ is always a 1-hyperseparator, so, $q_1 = 1$ and $m_{1,1} = 1$. $[A_{2,p_2}]$ can be either a 1-, 2- or (1, 1, ... , 1, 2)-hyperseparator (with any number of 1's), so, $m_{2,j} = 1$ (for all $1 \le j < q_2$, if $q_2 \ge 2$), $m_{2,q_2} = 2$ (if $q_2 \ge 2$) and $m_{2,q_2} = 1$ or 2 (if $q_2 = 1$). $[A_{3,p_3}]$ can be either a 1-, 2- or (1, 1, ... , 1, 2)-hyperseparator (with any number of 1's) but it can also be a 3-hyperseparator (if $[A_{2,p_2}]$ is a 2-hyperseparator), or a (1, 1, ... , 1, 2, 1, ... , 1, 2)- or (1, 1, ... , 1, 3)-hyperseparator (with q_2 entries in hyperlevel if $[A_{2,p_2}]$ is a (1, 1, ... , 1, 2)-hyperseparator (with q_2-1 1's or q_2 entries in hyperlevel, $q_2 \ge 2$)); $m_{3,j}$ can be 2 for up to two values of j between 1 and q_2 (including q_2 itself), or 3 for j = q_2 if $q_3 = q_2$ and $m_{2,q_2} = 2$. If $[A_{3,p_3}]$ is a (1, 1, ... , 1, 2, 1, ... , 1, 2)-hyperseparator (with 2's in the nth and q_3 (final) entries), the possible hyperlevels of $[A_{4,p_4}]$ would be as follows:

1,	
2,	
(1, 1, ... , 1, 2)	(any number of entries or 1's),
(1, 1, ... , 1, 2, 1, ... , 1, 2)	(2's in the nth and q_3th entries),
(1, 1, ... , 1, 2, 1, ... , 1, 2, 1, ... , 1, 2)	(2's in the kth, nth and q_3th entries, k < n),
(1, 1, ... , 1, 3, 1, ... , 1, 2)	(3 in the nth, 2 in the q_3th entries),
(1, 1, ... , 1, 3)	(3 in the q_3th entry),

with 1's making up the remaining entries. When going through the sequence of $[A_{i,p_i}]$ separators in reverse order from an (m_1, m_2, ... , m_q)-hyperseparator, and taking note of those of lower hyperlevels, we always pass through all of the (j, m_2, ... , m_q)-hyperseparators from j = m_1-1 down to j = 1, then all of the (1, j, m_3, ... , m_q)-hyperseparators from j = m_2-1 down to j = 1, then all of the (1, 1, j, m_4, ... , m_q)-hyperseparators from j = m_3-1 down to j = 1, and so on, to all of the (1, 1, ... , 1, j)-hyperseparators from j = m_q-1 down to j = 1.

The slash in Rule A5a is now the generalised $/_{m_1,m_2,\ldots,m_q}$ symbol, where $q \ge 1$ and $m_q \ge 2$ (if $q \ge 2$). Consequently, the $/_{m,m^*}$ in the $R_{n,i}$ equation is replaced by $/_{m_1,m_2,\ldots,m_q}$. The h subscript at the end of the $\#_s$ string becomes a subscript array H = 'h_1,h_2,...' and, in the case where $m_1 = m_2 = \ldots = m_k = 1$ and $m_{k+1} \ge 2$ for some positive integer k (previously m = 1 and m* ≥ 2), the h+b-n subscript at the end of the $\#_{n,s}$ string (1 < n < b) becomes the subscript array H but with the kth subscript being h_k+b-n instead of h_k. Rule A5a with $[A_{i,p_i}] = /_{1,1,\ldots,1,m_{k+1},m_{k+2},\ldots,m_q}$ ($m_{k+1} \ge 2$, q > k) means that at least one (1, 1, ... , 1, m_{k+1}, m_{k+2}, ... , m_q)-hyperseparator would be contained within the $R_{n,s}$ function, which would involve nesting through a series of (h_1, h_2, ... , h_{k-1}, h_k+b-n, m_{k+1}-1, m_{k+2}, m_{k+3}, ... , m_q)-hyperseparator angle brackets.

Rules A5a and A5a* are modified as follows:-

Rule A5a (separator $[A_{i,p_i}] = /_{m_1,m_2,\ldots,m_q}$, where $q \ge 1$ and $m_q \ge 2$ ($q \ge 2$)):
Set r to 1. For each j from 1 to i-1, increment r by 1 when either $q_j < q$ or $q_j = q$ and there exists a number n such that $m_{j,n} < m_n$ and $m_{j,k} = m_k$ (for all n < k ≤ q).

s = i-t_r,	
H = EndSub('$\#_s$')	(subscript array at the end of $\#_s$, this is 1 by default),
h_k = Sub(H, k)	(kth subscript of subscript array H = 'h_1,h_2,...'),
$\#^*_s$ = DelEndSub('$\#_s$')	(identical to $\#_s$ but with the end subscript array (H) deleted),
$\#_{n,s}$ = '$\#^*_{s\ h_1,h_2,\ldots,h_{k-1},(h_k+b-n),h_{k+1},h_{k+2},\ldots}$'	
	(1 < n < b, $m_j = 1$ (for all 1 ≤ j ≤ k), $m_{k+1} \ge 2$),
$\#_{n,k}$ = '$\#_k$'	(1 < n ≤ b, s ≤ k ≤ i, except k = s, $m_1 = 1$, q ≥ 2),
S_i = '$R_{b,i}$'.	

For $1 < n \le b$ and $s \le k < i$,

$R_{n,i} = $ ‘$b \langle A_{i,1}' \rangle\ b\ [A_{i,1}]\ b \langle A_{i,2}' \rangle\ b\ [A_{i,2}] \ldots b \langle A_{i,p_i-1}' \rangle\ b\ [A_{i,p_i-1}]\ b \langle R_{n-1,s} \rangle\ b\ /_{m_1,m_2,\ldots,m_q}\ c_i\text{-}1\ \#_{n,i}$’,

$R_{n,k} = $ ‘$b \langle A_{k,1}' \rangle\ b\ [A_{k,1}]\ b \langle A_{k,2}' \rangle\ b\ [A_{k,2}] \ldots b \langle A_{k,p_k-1}' \rangle\ b\ [A_{k,p_k-1}]\ b \langle R_{n,k+1} \rangle\ b\ [A_{k,p_k}]\ c_k\text{-}1\ \#_{n,k}$’,

$R_{1,s} = $ ‘0’.

Rule A5a* (separator $[A_{i,p_i}] = [d\ \#_{H\ m_1,m_2,\ldots}]$, where $d \ge 2$ and $\#_H$ contains a $(1, 1, \ldots , 1, r_1, r_2, \ldots)$-hyperseparator (with k 1’s) as the highest order hyperseparator in its base layer, where $k \ge 1$ and $r_1 \ge 2$):

$S_i = $ ‘$b \langle A_{i,1}' \rangle\ b\ [A_{i,1}]\ b \langle A_{i,2}' \rangle\ b\ [A_{i,2}] \ldots b \langle A_{i,p_i-1}' \rangle\ b\ [A_{i,p_i-1}]\ R_b\ [d\ \#_{H\ m_1,m_2,\ldots}]\ c_i\text{-}1\ \#_i$’,

$R_n = $ ‘$b \langle R_{n-1} \rangle\ b$’ $(n > 1)$,

$R_1 = $ ‘$b\ [d\text{-}1\ \#_{H\ 1,\ldots,1,m_k+b-1}]\ b$’ (with k-1 1’s in subscript).

In Rule A5a*, any of the m_j for $1 \le j \le k$ and r_j for $j \ge 2$ may take the value 1 ($m_j = 1$ for all $j > k$). Subscripts and hyperlevels are written with trailing 1’s removed. For example, if $n_k \ge 2$ but $n_i = 1$ for all $i > k$, then $/_{n_1,n_2,\ldots} = /_{n_1,n_2,\ldots,n_k}$, $[X_{\ n_1,n_2,\ldots}] = [X_{\ n_1,n_2,\ldots,n_k}]$ and $\langle X_{\ n_1,n_2,\ldots} \rangle = \langle X_{\ n_1,n_2,\ldots,n_k} \rangle$ for a string X, and an $(n_1, n_2, \ldots)$-hyperseparator would be an $(n_1, n_2, \ldots , n_k)$-hyperseparator. If $n_i = 1$ for all i, then $/_{n_1,n_2,\ldots} = /$, $[X_{\ n_1,n_2,\ldots}] = [X]$ and $\langle X_{\ n_1,n_2,\ldots} \rangle = \langle X \rangle$ for a string X, and an $(n_1, n_2, \ldots)$-hyperseparator would be a 1-hyperseparator.

Note that Rule A5a* with $\#_H$ containing the lowest $(1, 1, \ldots , 1, r_1, r_2, \ldots)$-hyperseparator (with k 1’s), as follows,

$[A_{i,p_i}] = [2\ /_{1,1,\ldots,1,r_1,r_2,\ldots}\ 2_{\ m_1,m_2,\ldots}]$ (with k 1’s after slash and $m_j = 1$ for all $j > k$)

would mean that

$R_1 = $ ‘$b\ [1\ /_{1,1,\ldots,1,r_1,r_2,\ldots}\ 2_{\ 1,\ldots,1,m_k+b-1}]\ b$’ (with k 1’s on left subscript and k-1 1’s on right)

$=$ ‘$b\ /_{1,\ldots,1,m_k+b-1,r_1-1,r_2,\ldots}\ b$’ (with k-1 1’s after slash),

which is how the (k+1)th subscript of the slash symbol is reduced by 1. (The kth subscript becomes m_k+b-1; all other subscripts remain unchanged.)

Rule A3 is changed as follows:-

Rule A3 (last entry in any 1-space or higher dimensional space of array is 1):

‘$a \langle \#\ [A]\ 1_{\ n_1,n_2,\ldots,n_k} \rangle\ b$’ = ‘$a \langle \#_{\ n_1,n_2,\ldots,n_k} \rangle\ b$’.

When [A] is an $(m_1, m_2, \ldots , m_p)$-hyperseparator, [B] is an $(n_1, n_2, \ldots , n_q)$-hyperseparator and either $p < q$; $p = q$ and there exists a number k such that $m_k < n_k$ and $m_i = n_i$ (for all $k < i \le p$); or $p = q$, $m_i = n_i$ (for all $1 \le i \le p$) and level of [A] is less than level of [B],

‘$a \langle \#\ [A]\ 1\ [B]\ \#^* \rangle\ b$’ = ‘$a \langle \#\ [B]\ \#^* \rangle\ b$’.

Remove trailing 1’s.

Subscript arrays can themselves be extended into multidimensional arrays, nested arrays or even as advanced as ordinary separator arrays can go. The single slash without any subscripts is now equivalent to

$[1\ /_{1\ [2]\ 2}\ 2] = [1\ /_{1\ [2]\ 1,2}\ 2] = [1\ /_{1\ [2]\ 1,1,2}\ 2] = \ldots$,

$[1\ /_{1\ [2]\ 1\ [2]\ 2}\ 2] = [1\ /_{1\ [2]\ 1\ [2]\ 1,2}\ 2] = [1\ /_{1\ [2]\ 1\ [2]\ 1,1,2}\ 2] = \ldots$,

$[1\ /_{1\ [2]\ 1\ [2]\ 1\ [2]\ 2}\ 2] = [1\ /_{1\ [2]\ 1\ [2]\ 1\ [2]\ 1\ [2]\ 2}\ 2] = [1\ /_{1\ [2]\ 1\ [2]\ 1\ [2]\ 1\ [2]\ 1\ [2]\ 2}\ 2] = \ldots$,

$[1\ /_{1\ [3]\ 2}\ 2] = [1\ /_{1\ [3]\ 1,2}\ 2] = \ldots = [1\ /_{1\ [3]\ 1\ [2]\ 2}\ 2] = [1\ /_{1\ [3]\ 1\ [2]\ 1,2}\ 2] = \ldots$,

$[1\ /_{1\ [3]\ 1\ [3]\ 2}\ 2] = [1\ /_{1\ [3]\ 1\ [3]\ 1\ [3]\ 2}\ 2] = [1\ /_{1\ [3]\ 1\ [3]\ 1\ [3]\ 1\ [3]\ 2}\ 2] = \ldots$,

$[1\ /_{1\ [4]\ 2}\ 2] = [1\ /_{1\ [5]\ 2}\ 2] = [1\ /_{1\ [6]\ 2}\ 2] = \ldots$,

$[1\ /_{1\ [1,2]\ 2}\ 2] = [1\ /_{1\ [1\ [2]\ 2]\ 2}\ 2] = [1\ /_{1\ [1\ [1,2]\ 2]\ 2}\ 2] = \ldots$,

$[1\ /_{1\ [1\ /\ 2]\ 2}\ 2] = [1\ /_{1\ [1\ /\ 3]\ 2}\ 2] = [1\ /_{1\ [1\ /\ 4]\ 2}\ 2] = \ldots$,

$[1\ /_{1\ [1\ /\ 1\ /\ 2]\ 2}\ 2] = [1\ /_{1\ [1\ /\ 1\ /\ 1\ /\ 2]\ 2}\ 2] = [1\ /_{1\ [1\ /\ 1\ /\ 1\ /\ 1\ /\ 2]\ 2}\ 2] = \ldots$,

$[1\ /_{1\ [1\ [1\ /\ 2\ /_2\ 2]\ 2]\ 2}\ 2] = [1\ /_{1\ [1\ [1\ /\ 1\ /\ 2\ /_2\ 2]\ 2]\ 2}\ 2] = [1\ /_{1\ [1\ [1\ [1\ /\ 2\ /_2\ 2]\ 2\ /_2\ 2]\ 2]\ 2}\ 2] = \ldots$,

$[1\ /_{1\ [1\ [1\ [1\ /_2\ 2\ /_3\ 2]\ 2]\ 2]\ 2}\ 2] = [1\ /_{1\ [1\ [1\ [1\ [1\ /_3\ 2\ /_4\ 2]\ 2]\ 2]\ 2]\ 2}\ 2] = [1\ /_{1\ [1\ [1\ [1\ [1\ [1\ /_4\ 2\ /_5\ 2]\ 2]\ 2]\ 2]\ 2]\ 2}\ 2] = \ldots$,

$[1\ /_{1\ [1\ [2\ /_{1,2}\ 2]\ 2]\ 2}\ 2] = [1\ /_{1\ [1\ [2\ /_{1,1,2}\ 2]\ 2]\ 2}\ 2] = [1\ /_{1\ [1\ [2\ /_{1,1,1,2}\ 2]\ 2]\ 2}\ 2] = \ldots$,

$[1\ /_{S_1}\ 2] = [1\ /_{S_2}\ 2] = [1\ /_{S_3}\ 2] = \ldots$

(with S_1 = ‘$1\ [1\ [2\ /_{1,2}\ 2]\ 2]\ 2$’ and S_{n+1} = ‘$1\ [1\ [2\ /_{S_n}\ 2]\ 2]\ 2$’).

In fact,

$/ = [1\ /_{1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_k]\ 2}\ 2]$,

in all cases where each $[A_i]$ is a normal separator. A subscript array, like a main array (in curly brackets), can only contain normal separators in its ‘base layer’.

According to the Taxicab Rule, an $(n_1\ [A_1]\ n_2\ [A_2]\ \ldots\ [A_k]\ n_{k+1})$-hyperseparator (the lowest of which is the $/_{n_1\ [A_1]\ n_2\ [A_2]\ \ldots\ [A_k]\ n_{k+1}}$ symbol), where each $[A_i]$ is a normal separator, requires a minimum of $n_1+n_2+\ldots+n_{k+1}-k$ pairs of square brackets in order to turn it into a normal separator. This holds true when every $[A_i] = [1]$ (comma).

The generalised slash subscript array (for normal separators $[A_i]$)

$/_{n_1\ [A_1]\ n_2\ [A_2]\ \ldots\ [A_k]\ n_{k+1}}$

$= [1\ \ /_{n_1+1\ [A_1]\ n_2\ [A_2]\ n_3\ [A_3]\ n_4\ [A_4]\ \ldots\ [A_k]\ n_{k+1}}\ \ 2]$

$= [1\ \ /_{1\ [A_1]\ n_2+1\ [A_2]\ n_3\ [A_3]\ n_4\ [A_4]\ \ldots\ [A_k]\ n_{k+1}}\ \ 2\ \ _{n_1}]$

$= [1\ \ /_{1\ [A_1]\ 1\ [A_2]\ n_3+1\ [A_3]\ n_4\ [A_4]\ \ldots\ [A_k]\ n_{k+1}}\ \ 2\ \ _{n_1\ [A_1]\ n_2}]$

$= \ldots\ldots$

$= [1\ \ /_{1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_k]\ n_{k+1}+1}\ \ 2\ \ _{n_1\ [A_1]\ n_2\ [A_2]\ \ldots\ [A_{k-1}]\ n_k}]$

$= [1\ \ /_{S\ [A_1]\ n_2\ [A_2]\ n_3\ [A_3]\ n_4\ [A_4]\ \ldots\ [A_k]\ n_{k+1}}\ \ 2\ \ _{n_1}]$ $([B_1] < [A_1])$

$= [1\ \ /_{1\ [A_1]\ S\ [A_2]\ n_3\ [A_3]\ n_4\ [A_4]\ldots\ [A_k]\ n_{k+1}}\ \ 2\ \ _{n_1\ [A_1]\ n_2}]$ $([B_1] < [A_2])$

$= [1\ \ /_{1\ [A_1]\ 1\ [A_2]\ S\ [A_3]\ n_4\ [A_4]\ldots\ [A_k]\ n_{k+1}}\ \ 2\ \ _{n_1\ [A_1]\ n_2\ [A_2]\ n_3}]$ $([B_1] < [A_3])$

$= \ldots\ldots$

$= [1\ \ /_{1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_{k-1}]\ S\ [A_k]\ n_{k+1}}\ \ 2\ \ _{n_1\ [A_1]\ n_2\ [A_2]\ \ldots\ [A_{k-1}]\ n_k}]$ $([B_1] < [A_k])$

$= [1\ \ /_{1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_k]\ S}\ \ 2\ \ _{n_1\ [A_1]\ n_2\ [A_2]\ \ldots\ [A_k]\ n_{k+1}}]$,

where S = ‘$1\ [B_1]\ 1\ [B_2]\ \ldots\ 1\ [B_m]\ 2$’ ($m \geq 1$, $[B_i]$ are normal separators).

The slash itself in the above equalities (other than the last one) may be substituted by a separator array that contains at least one $(1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_k]\ 1\ \#)$-hyperseparator (with # non-empty, containing at least one entry of 2 or greater) in its ‘base layer’, for example,

$[X\ \ /_{1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_k]\ 1,m}\ \ Y\ \ _{n_1\ [A_1]\ n_2\ [A_2]\ \ldots\ [A_k]\ n_{k+1}}]$

$= [1\ \ [X\ \ /_{1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_k]\ 1,m}\ \ Y\ \ _{1\ [A_1]\ 1\ [A_2]\ n_3+1\ [A_3]\ n_4\ [A_4]\ \ldots\ [A_k]\ n_{k+1}}]\ \ 2\ \ _{n_1\ [A_1]\ n_2}]$

(with $m \geq 2$; X and Y are strings either side of $/_{1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_k]\ 1,m}$).

The $\theta(\Omega_{\omega\text{^}\omega})$ level separator

$\{a, b\ [1\ [2\ /_{1\ [2]\ 2}\ 2]\ 2]\ 2\} = \{a\ \langle 0\ [2\ /_{1\ [2]\ 2}\ 2]\ 2\rangle\ b\}$

$= \{a\ \langle b\ \langle b\ \langle\ \ldots\ \langle b\ \langle b\ /_{1,\ldots,1,b}\ b\rangle\ b\rangle\ \ldots\ \rangle\ b\rangle\ b\rangle\ b\}$

(with b pairs of angle brackets and b-1 1’s in $/_{1,\ldots,1,b}$).

This is equivalent to

$\{a, b\ [1\ [2\ /_{1,1,\ldots,1,2}\ 2]\ 2]\ 2\}$ (with b 1’s in $/_{1,1,\ldots,1,2}$).

With k 1’s in the subscript,

$\{a, b\ [1\ [1\ [2\ /_{1,1,\ldots,1,2\ [2]\ 2}\ 2]\ 2]\ 2]\ 2\} = \{a\ \langle 0\ [1\ [2\ /_{1,1,\ldots,1,2\ [2]\ 2}\ 2]\ 2]\ 2\rangle\ b\}$

$= \{a\ \langle b\ \langle b\ \langle\ \ldots\ \langle b\ \langle b\ /_{1,\ldots,1,b\ [2]\ 2}\ b\rangle\ b\rangle\ \ldots\ \rangle\ b\rangle\ b\rangle\ b\}$

(with b+1 pairs of angle brackets and k-1 1’s in $_{1,\ldots,1,b}$),

$\{a, b\ [1\ [2\ /_{1\ [2]\ 1,1,\ldots,1,2}\ 2]\ 2]\ 2\} = \{a\ \langle 0\ [2\ /_{1\ [2]\ 1,1,\ldots,1,2}\ 2]\ 2\rangle\ b\}$

$= \{a \langle b \langle b \langle \dots \langle b \langle b\ /_{1\,[2]\,1,\dots,1,b}\ b\rangle\ b\rangle \dots \rangle\ b\rangle\ b\rangle\ b\}$
(with b pairs of angle brackets and k-1 1's in $_{1,\dots,1,b}$).

If there was a chain of 1's and [2]'s prior to the $_{1,1,\dots,1,2}$ in the subscripts of the above two arrays, this chain would remain unchanged prior to the $_{1,\dots,1,b}$ in the subscripts on the right-hand side.

When X is a subscript array beginning with 1 and containing at least one entry of 2 or greater, and n is one more than the taxicab distance of X (the sum of all entries in its 'base layer' when reduced by 1),

$\{a, b\ [1\ [1\ [1 \dots [1\ [c\ /_X\ 2]\ 2] \dots 2]\ 2]\ 2]\ 2\}$ (with n pairs of square brackets)
$= \{a \langle 0\ [1\ [1 \dots [1\ [c\ /_X\ 2]\ 2] \dots 2]\ 2]\ 2\rangle\ b\}$
$= \{a \langle b \langle b \langle \dots \langle b \langle b\ [c\text{-}1\ /_X\ 2\ _Y]\ b\rangle\ b\rangle \dots \rangle\ b\rangle\ b\rangle\ b\}$ (with n+b-2 pairs of angle brackets),

for some other subscript array Y (where [X] > [Y]). If c = 2, the separator $[1\ /_X\ 2\ _Y]$ 'drops down' to $/_Z$ (where Z is another subscript array) since the first non-1 entry in X is beyond the entry corresponding to the final non-1 entry in Y. Z would inherit all the entries and separators of Y plus the entries and separators of X from the first separator in X that has no counterpart in Y, onwards (with the first non-1 entry of X reduced by 1). This implies that Y and Z would be identical when X contains a single non-1 entry of 2.

I find that

$[1\ \ [2\ /_{1\,[2]\,2}\ 2]\ \ 2]$ has level $\theta(\Omega_{\omega^\wedge\omega})$,
$[1\ \ [2\ /_{1\,[2]\,2}\ 2]\ \ 3]$ has level $\theta(\theta_1(\Omega_{\omega^\wedge\omega}), 1)$,
$[1\ \ [2\ /_{1\,[2]\,2}\ 2]\ \ 1\ /\ 2]$ has level $\theta(\theta_1(\Omega_{\omega^\wedge\omega})+1)$,
$[1\ \ [2\ /_{1\,[2]\,2}\ 2]\ \ 1\ \ [2\ /_{1\,[2]\,2}\ 2]\ \ 2]$ has level $\theta(\theta_1(\Omega_{\omega^\wedge\omega})2)$,
$[1\ [1\ /_2\ 2\ \ [2\ /_{1\,[2]\,2}\ 2\ _2]\ \ 2]\ 2]$ has level $\theta(\varepsilon(\theta_1(\Omega_{\omega^\wedge\omega})+1))$,
$[1\ [1\ \ [2\ /_{1\,[2]\,2}\ 2\ _2]\ \ 3]\ 2]$ has level $\theta(\theta_2(\Omega_{\omega^\wedge\omega}), 1)$,
$[1\ [1\ [1\ \ [2\ /_{1\,[2]\,2}\ 2\ _3]\ \ 3]\ 2]\ 2]$ has level $\theta(\theta_3(\Omega_{\omega^\wedge\omega}), 1)$,
$[1\ [2\ \ [2\ /_{1\,[2]\,2}\ 2\ _{1,2}]\ \ 2]\ 2]$ has level $\theta(\theta_\omega(\Omega_{\omega^\wedge\omega}), 1)$ ($[2\ /_{1\,[2]\,2}\ 2] = [1\ [2\ /_{1\,[2]\,2}\ 2\ _{1,2}]\ 2]$),
$[1\ [1\ [2\ \ [2\ /_{1\,[2]\,2}\ 2\ _{1,3}]\ \ 2]\ 2]\ 2]$ has level $\theta(\theta_{\omega 2}(\Omega_{\omega^\wedge\omega}), 1)$,
$[1\ [1\ [1\ [2\ \ [2\ /_{1\,[2]\,2}\ 2\ _{1,4}]\ \ 2]\ 2]\ 2]\ 2]$ has level $\theta(\theta_{\omega 3}(\Omega_{\omega^\wedge\omega}), 1)$.

Since $[2\ /_{1\,[2]\,2}\ 2] = [1\ [2\ /_{1\,[2]\,2}\ 2\ _{1,1,\dots,1,2}]\ 2]$, for any finite number of 1's in $_{1,1,\dots,1,2}$,

$[1\ [2\ \ [2\ /_{1\,[2]\,2}\ 2\ _{1,1,2}]\ \ 2]\ 2]$ has level $\theta(\theta_{\omega^\wedge 2}(\Omega_{\omega^\wedge\omega}), 1)$,
$[1\ [2\ \ [2\ /_{1\,[2]\,2}\ 2\ _{1,1,\dots,1,2}]\ \ 2]\ 2]$ (with n 1's in $_{1,1,\dots,1,2}$) has level $\theta(\theta_{\omega^\wedge n}(\Omega_{\omega^\wedge\omega}), 1)$,
$[1\ [3\ /_{1\,[2]\,2}\ 2]\ 2]$ has level $\theta(\Omega_{\omega^\wedge\omega}, 1)$ (limit ordinal of above as n → ω),
$[1\ [1\ /\ 2\ /_{1\,[2]\,2}\ 2]\ 2]$ has level $\theta(\Omega_{\omega^\wedge\omega}, \Omega)$,
$[1\ [1\ /_{1,2}\ 2\ /_{1\,[2]\,2}\ 2]\ 2]$ has level $\theta(\Omega_{\omega^\wedge\omega}, \Omega_\omega)$,
$[1\ [1\ /_{1,1,\dots,1,2}\ 2\ /_{1\,[2]\,2}\ 2]\ 2]$ (with n 1's in $_{1,1,\dots,1,2}$) has level $\theta(\Omega_{\omega^\wedge\omega}, \Omega_{\omega^\wedge n})$,
$[1\ [1\ /_{1\,[2]\,2}\ 3]\ 2]$ has level $\theta(\Omega_{\omega^\wedge\omega}+1)$,
$[1\ [1\ /_{1\,[2]\,2}\ 1\ /_{1\,[2]\,2}\ 2]\ 2]$ has level $\theta(\Omega_{\omega^\wedge\omega}2)$,
$[1\ [1\ /_{1\,[2]\,2}\ 1\ /_{1\,[2]\,2}\ 1\ /_{1\,[2]\,2}\ 2]\ 2]$ has level $\theta(\Omega_{\omega^\wedge\omega}{}^\wedge 2)$,
$[1\ [1\ [2\ /_{2\,[2]\,2}\ 2]\ 2]\ 2]$ has level $\theta(\Omega_{\omega^\wedge\omega}{}^\wedge\omega)$ ($/_{n\,[2]\,2} = [1\ /_{n+1\,[2]\,2}\ 2]$),
$[1\ [1\ [1\ /_{1\,[2]\,2}\ 2\ /_{2\,[2]\,2}\ 2]\ 2]\ 2]$ has level $\theta(\Omega_{\omega^\wedge\omega}{}^\wedge\Omega_{\omega^\wedge\omega})$,
$[1\ [1\ [1\ /_{2\,[2]\,2}\ 3]\ 2]\ 2]$ has level $\theta(\varepsilon(\Omega_{\omega^\wedge\omega}+1)) = \theta(\theta_{\omega^\wedge\omega}(1))$,
$[1\ [1\ [1\ [1\ /_{2\,[2]\,2}\ 2\ /_{3\,[2]\,2}\ 2]\ 2]\ 2]\ 2]$ has level $\theta(\Omega_{\omega^\wedge\omega+1})$,
$[1\ [1\ [1\ [1\ [1\ /_{3\,[2]\,2}\ 2\ /_{4\,[2]\,2}\ 2]\ 2]\ 2]\ 2]\ 2]$ has level $\theta(\Omega_{\omega^\wedge\omega+2})$,
$[1\ [1\ [2\ /_{1,2\,[2]\,2}\ 2]\ 2]\ 2]$ has level $\theta(\Omega_{\omega^\wedge\omega+\omega})$ ($/_{1,n\,[2]\,2} = [1\ /_{1,n+1\,[2]\,2}\ 2]$),
$[1\ [1\ [1\ [2\ /_{1,3\,[2]\,2}\ 2]\ 2]\ 2]\ 2]$ has level $\theta(\Omega_{\omega^\wedge\omega+\omega 2})$,
$[1\ [1\ [2\ /_{1,1,2\,[2]\,2}\ 2]\ 2]\ 2]$ has level $\theta(\Omega_{\omega^\wedge\omega+\omega^\wedge 2})$ ($/_{1,1,n\,[2]\,2} = [1\ /_{1,1,n+1\,[2]\,2}\ 2]$),
$[1\ [1\ [2\ /_{1,1,\dots,1,2\,[2]\,2}\ 2]\ 2]\ 2]$ (with n 1's in $_{1,1,\dots,1,2}$) has level $\theta(\Omega_{\omega^\wedge\omega+\omega^\wedge n})$,
$[1\ [1\ [2\ /_{1\,[2]\,3}\ 2]\ 2]\ 2]$ has level $\theta(\Omega_{(\omega^\wedge\omega)2})$ ($/_{1\,[2]\,n} = [1\ /_{1\,[2]\,n+1}\ 2]$),

$[1\ [1\ [1\ [2\ /_{1\ [2]\ 4}\ 2]\ 2]\ 2]\ 2]$ has level $\theta(\Omega_{(\omega\text{^}\omega)3})$,
$[1\ [2\ /_{1\ [2]\ 1,2}\ 2]\ 2]$ has level $\theta(\Omega_{\omega\text{^}(\omega+1)})$,
$[1\ [2\ /_{1\ [2]\ 1,1,\ldots,1,2}\ 2]\ 2]$ (with n 1's in $_{1,1,\ldots,1,2}$) has level $\theta(\Omega_{\omega\text{^}(\omega+n)})$,
$[1\ [2\ /_{1\ [2]\ 1\ [2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\omega\text{^}(\omega 2)})$,
$[1\ [2\ /_{1\ [2]\ 1\ [2]\ 1\ [2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\omega\text{^}(\omega 3)})$,
$[1\ [2\ /_{1\ [2]\ 1\ [2]\ \ldots\ 1\ [2]\ 2}\ 2]\ 2]$ (with n [2]'s) has level $\theta(\Omega_{\omega\text{^}(\omega n)})$.

The most significant higher separators are as follows:-
$[1\ [2\ /_{1\ [3]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\omega\text{^}\omega\text{^}2})$,
$[1\ [2\ /_{1\ [4]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\omega\text{^}\omega\text{^}3})$,
$[1\ [2\ /_{1\ [1,2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\omega\text{^}\omega\text{^}\omega})$,
$[1\ [2\ /_{1\ [1\ [2]\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\omega\text{^}\omega\text{^}\omega\text{^}\omega})$,
$[1\ [2\ /_{1\ [1\ [1,2]\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\omega\text{^}\omega\text{^}\omega\text{^}\omega\text{^}\omega})$,
$[1\ [2\ /_{1\ [1\ /\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\varepsilon_0}) = \theta(\Omega_{\theta(1)})$,
$[1\ [2\ /_{1\ [1\ /\ 3]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\varepsilon_1}) = \theta(\Omega_{\theta(1,\ 1)})$,
$[1\ [2\ /_{1\ [1\ /\ 1\ /\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\zeta_0}) = \theta(\Omega_{\theta(2)})$,
$[1\ [2\ /_{1\ [1\ /\ 1\ /\ 1\ /\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\theta(3)})$,
$[1\ [2\ /_{1\ [1\ [2\ /_2\ 2]\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\theta(\omega)})$,
$[1\ [2\ /_{1\ [1\ [1\ /\ 2\ /_2\ 2]\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\Gamma_0}) = \theta(\Omega_{\theta(\Omega)})$,
$[1\ [2\ /_{1\ [1\ [1\ /\ 1\ /\ 2\ /_2\ 2]\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\theta(\Omega\text{^}\Omega)})$,
$[1\ [2\ /_{1\ [1\ [1\ [1\ /\ 2\ /_2\ 2]\ 2\ /_2\ 2]\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\theta(\Omega\text{^}\Omega\text{^}\Omega)})$,
$[1\ [2\ /_{1\ [1\ [1\ /_2\ 3]\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\theta(\varepsilon_{\Omega+1})}) = \theta(\Omega_{\theta(\theta_1(1))})$,
$[1\ [2\ /_{1\ [1\ [1\ /_2\ 1\ /_2\ 2]\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\theta(\zeta_{\Omega+1})}) = \theta(\Omega_{\theta(\theta_1(2))})$,
$[1\ [2\ /_{1\ [1\ [1\ [2\ /_3\ 2]\ 2]\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\theta(\theta_1(\omega))})$,
$[1\ [2\ /_{1\ [1\ [1\ [1\ /_2\ 2\ /_3\ 2]\ 2]\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\theta(\Omega_2)})$,
$[1\ [2\ /_{1\ [1\ [1\ [1\ [1\ /_3\ 2\ /_4\ 2]\ 2]\ 2]\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\theta(\Omega_3)})$,
$[1\ [2\ /_{1\ [1\ [2\ /_{1,2}\ 2]\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\theta(\Omega_\omega)})$,
$[1\ [2\ /_{1\ [1\ [1\ [2\ /_{1,3}\ 2]\ 2]\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\theta(\Omega_{\omega 2})})$,
$[1\ [2\ /_{1\ [1\ [2\ /_{1,1,2}\ 2]\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\theta(\Omega_{\omega\text{^}2})})$,
$[1\ [2\ /_{1\ [1\ [2\ /_{1,1,1,2}\ 2]\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\theta(\Omega_{\omega\text{^}3})})$,
$[1\ [2\ /_{1\ [1\ [2\ /_{1\ [2]\ 2}\ 2]\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\theta(\Omega_{\omega\text{^}\omega})})$,
$[1\ [2\ /_{1\ [1\ [2\ /_{1\ [1\ /\ 2]\ 2}\ 2]\ 2]\ 2}\ 2]\ 2]$ has level $\theta(\Omega_{\theta(\Omega_{\varepsilon_0})})$.

Taking S_1 = '1 [1 [2 $/_{1,2}$ 2] 2] 2' and S_{n+1} = '1 [1 [2 $/_{S_n}$ 2] 2] 2',
$[1\ [2\ /_{S_1}\ 2]\ 2]$ has level $\theta(\Omega_{\theta(\Omega_\omega)})$,
$[1\ [2\ /_{S_2}\ 2]\ 2]$ has level $\theta(\Omega_{\theta(\Omega_{\theta(\Omega_\omega)})})$,
$[1\ [2\ /_{S_3}\ 2]\ 2]$ has level $\theta(\Omega_{\theta(\Omega_{\theta(\Omega_{\theta(\Omega_\omega)})})})$.

The limit ordinal of the S_n sequence, and the nested slash subscript array notation – which I will call the Nested Hierarchical Hyper-Nested Array Notation – is $\theta(\Omega_\Omega)$. This is an ordinal so huge that even the level of the uncountable ordinal is itself uncountable!

The $\theta(\Omega_{\omega\text{^}\omega\text{^}2})$ level separator

$\{a,\ b\ [1\ [2\ /_{1\ [3]\ 2}\ 2]\ 2]\ 2\} = \{a\ \langle 0\ [2\ /_{1\ [3]\ 2}\ 2]\ 2\rangle\ b\}$
$= \{a\ \langle b\ \langle b\ \langle\ \ldots\ \langle b\ \langle b\ /_{1\ \langle 2\rangle\ b\ (\leftarrow b)}\ b\rangle\ b\rangle\ \ldots\ \rangle\ b\rangle\ b\rangle\ b\}$
(with b pairs of angle brackets outside subscript),

where the $(\leftarrow b)$ means replace the final entry by b. Since

'1 ‹2› b (←b)' = '1 ‹1› b [2] 1 ‹1› b [2] ... 1 ‹1› b [2] 1 ‹1› b (←b)' (with b-1 [2]'s)
= '1,1,..,1 [2] 1,1,..,1 [2] .... 1,1,..,1 [2] 1,1,..,1,b' (b-1 1's after final [2])
= '1 [2] 1 [2] ... 1 [2] 1,1,...,1,b' (remove trailing 1's),

it follows that

{a, b [1 [2 $/_{1\,[3]\,2}$ 2] 2] 2} = {a ‹b ‹b ‹ ... ‹b ‹b $/_{1\,[2]\,1\,[2]\,\ldots\,1\,[2]\,1,1,\ldots,1,b}$ b› b› ... › b› b› b}
(b pairs of angle brackets, b-1 ‘1 [2]’s and b-1 1’s in $_{1,1,\ldots,1,b}$).

In the case of the $\theta(\Omega_{\omega\text{^}\omega})$ level separator in {a, b [1 [2 $/_{1\,[2]\,2}$ 2] 2] 2}, the slash subscript array between the pair of b’s inside the innermost angle brackets on the right-hand side would be $/_{1\,\langle 1\rangle\,b\,(\leftarrow b)}$ as this gives $/_{1,1,\ldots,1,b}$ with b-1 1’s. The introduction of the ‘left arrow’ in Rule A5a* means that a few modifications to Rules A1, A2 and A4 are necessary in order to get rid of the arrow when the angle brackets disappear.

The following additions are made to Rule A1:

‘a ‹0› b (←c)’ = ‘c’,
‘a ‹1› b (←c)’ = ‘a, a, ... , a, c’ (with b-1 a’s).

The following additions are made to Rule A2:

‘a ‹0 #› b (←c)’ = ‘c’,
‘a ‹1 #› b (←c)’ = ‘a [1 #] a [1 #] ... a [1 #] c’ (with b-1 a’s),
where # begins with a 2- or higher order hyperseparator.

The following addition is made to Rule A4:

‘a ‹A› 1 (←c)’ = ‘c’.

The $\theta(\Omega_{\omega\text{^}\omega\text{^}\omega})$ level separator

{a, b [1 [2 $/_{1\,[1,2]\,2}$ 2] 2] 2} = {a ‹0 [2 $/_{1\,[1,2]\,2}$ 2] 2› b}
= {a ‹b ‹b ‹ ... ‹b ‹b $/_{1\,\langle 0,2\rangle\,b\,(\leftarrow b)}$ b› b› ... › b› b› b}
= {a ‹b ‹b ‹ ... ‹b ‹b $/_{1\,\langle b\rangle\,b\,(\leftarrow b)}$ b› b› ... › b› b› b}
(with b pairs of angle brackets outside subscript),
where ‘1 ‹b› b (←b)’ = ‘1 ‹b-1› b [b] 1 ‹b-1› b [b] ... 1 ‹b-1› b [b] 1 ‹b-1› b (←b)’ (with b-1 [b]’s)
= ‘1 [b] 1 [b] ... 1 [b] 1 [b-1] 1 [b-1] ... 1 [b-1] ...... 1 [2] 1 [2] ... 1 [2] 1,1,...,1,b’
(b-1 each of [b]’s, [b-1]’s, ... , [2]’s and b-1 1’s after final [2]).

Examples of arrays with separators of higher levels are as follows:-

{a, b [1 [2 $/_{1\,[1\,/\,2]\,2}$ 2] 2] 2}
= {a ‹0 [2 $/_{1\,[1\,/\,2]\,2}$ 2] 2› b}
= {a ‹b ‹b ‹ ... ‹b ‹b $/_{1\,\langle 0\,/\,2\rangle\,b\,(\leftarrow b)}$ b› b› ... › b› b› b}
(with b pairs of angle brackets outside subscript)
= {a ‹b ‹b ‹ ... ‹b ‹b $/_{1\,\langle b\,\langle b\,\langle\,\ldots\,\langle b\,\langle b\rangle\,b\rangle\,\ldots\,\rangle\,b\rangle\,b\rangle\,b\,(\leftarrow b)}$ b› b› ... › b› b› b}
(with b-1 pairs of angle brackets in subscript),

{a, b [1 [2 $/_{1\,[1\,[1\,/_2\,3]\,2]\,2}$ 2] 2] 2}
= {a ‹0 [2 $/_{1\,[1\,[1\,/_2\,3]\,2]\,2}$ 2] 2› b}
= {a ‹b ‹b ‹ ... ‹b ‹b $/_{1\,\langle 0\,[1\,/_2\,3]\,2\rangle\,b\,(\leftarrow b)}$ b› b› ... › b› b› b}
(with b pairs of angle brackets outside subscript)
= {a ‹b ‹b ‹ ... ‹b ‹b $/_{1\,\langle b\,\langle b\,\langle b\,\langle\,\ldots\,\langle b\,\langle b\,/_2\,2\rangle\,b\,/_2\,2\rangle\,\ldots\,\rangle\,b\,/_2\,2\rangle\,b\,/_2\,2\rangle\,b\rangle\,b\,(\leftarrow b)}$ b› b› ... › b› b› b}
(with b pairs of angle brackets in subscript),

{a, b [1 [2 $/_{1\,[1\,[2\,/_{1,2}\,2]\,2]\,2}$ 2] 2] 2}
= {a ‹0 [2 $/_{1\,[1\,[2\,/_{1,2}\,2]\,2]\,2}$ 2] 2› b}
= {a ‹b ‹b ‹ ... ‹b ‹b $/_{1\,\langle 0\,[2\,/_{1,2}\,2]\,2\rangle\,b\,(\leftarrow b)}$ b› b› ... › b› b› b}
(with b pairs of angle brackets outside subscript)
= {a ‹b ‹b ‹ ... ‹b ‹b $/_{1\,\langle b\,\langle b\,\langle\,\ldots\,\langle b\,\langle b\,/_b\,b\rangle\,b\rangle\,\ldots\,\rangle\,b\rangle\,b\rangle\,b\,(\leftarrow b)}$ b› b› ... › b› b› b}

(with b pairs of angle brackets in subscript).

In general, when each $[A_i]$ is a normal separator,

$\{a, b\ [1\ [2\ /_{1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_k]\ 2}\ 2]\ 2]\ 2\}$

$= \{a \langle 0\ [2\ /_{1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_k]\ 2}\ 2]\ 2\rangle\ b\}$

$= \{a \langle b \langle b \langle \ldots \langle b \langle b\ /_{1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_{k-1}]\ 1\ \langle A_k' \rangle\ b\ (\leftarrow b)}\ b\rangle\ b\rangle\ \ldots\ \rangle\ b\rangle\ b\rangle\ b\}$

(with b pairs of angle brackets and k 1's),

where A_k' is identical to A_k except that the first entry is reduced by 1.

The 'left arrow' is also needed for the following separators which require a revised Rule A5a (the first of which has level $\theta(\Omega_{\omega\text{^}\omega}+1)$), though no $R_{n,k}$ string nesting operations are required:-

$\{a, b\ [1\ [1\ /_{1\ [2]\ 2}\ 3]\ 2]\ 2\} = \{a \langle 0\ [1\ /_{1\ [2]\ 2}\ 3]\ 2\rangle\ b\}$

$= \{a \langle b \langle b\ /_{1\ \langle 1\rangle\ b\ (\leftarrow 2)}\ 3\ /_{1\ [2]\ 2}\ 2\rangle\ b\rangle\ b\}$,

$\{a, b\ [1\ [1\ /_{1\ [2]\ 2}\ 1\ /_{1\ [2]\ 2}\ 2]\ 2]\ 2\} = \{a \langle 0\ [1\ /_{1\ [2]\ 2}\ 1\ /_{1\ [2]\ 2}\ 2]\ 2\rangle\ b\}$

$= \{a \langle b \langle b\ /_{1\ [2]\ 2}\ b\ /_{1\ \langle 1\rangle\ b\ (\leftarrow 2)}\ 2\rangle\ b\rangle\ b\}$,

$\{a, b\ [1\ [1\ [1\ /_{1\ [2]\ 3}\ 3]\ 2]\ 2]\ 2\} = \{a \langle 0\ [1\ [1\ /_{1\ [2]\ 3}\ 3]\ 2]\ 2\rangle\ b\}$

$= \{a \langle b \langle b \langle b\ /_{1\ \langle 1\rangle\ b\ (\leftarrow 2)\ [2]\ 2}\ 3\ /_{1\ [2]\ 3}\ 2\rangle\ b\rangle\ b\rangle\ b\}$,

$\{a, b\ [1\ [1\ /_{1\ [2]\ 1\ [2]\ 2}\ 3]\ 2]\ 2\} = \{a \langle 0\ [1\ /_{1\ [2]\ 1\ [2]\ 2}\ 3]\ 2\rangle\ b\}$

$= \{a \langle b \langle b\ /_{1\ [2]\ 1\ \langle 1\rangle\ b\ (\leftarrow 2)}\ 3\ /_{1\ [2]\ 1\ [2]\ 2}\ 2\rangle\ b\rangle\ b\}$,

$\{a, b\ [1\ [1\ /_{1\ [3]\ 2}\ 3]\ 2]\ 2\} = \{a \langle 0\ [1\ /_{1\ [3]\ 2}\ 3]\ 2\rangle\ b\}$

$= \{a \langle b \langle b\ /_{1\ \langle 2\rangle\ b\ (\leftarrow 2)}\ 3\ /_{1\ [3]\ 2}\ 2\rangle\ b\rangle\ b\}$,

where '$1 \langle 1\rangle\ b\ (\leftarrow 2)$' = '1,1,...,1,2' (with b-1 1's),

'$1 \langle 2\rangle\ b\ (\leftarrow 2)$' = '1 [2] 1 [2] ... 1 [2] 1,1,...,1,2' (with b-1 [2]'s and b-1 1's after final [2]).

In general, when c ≥ 3 and each $[A_i]$ is a normal separator,

$\{a, b\ [1\ [1\ /_{1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_k]\ 2}\ c]\ 2]\ 2\}$

$= \{a \langle 0\ [1\ /_{1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_k]\ 2}\ c]\ 2\rangle\ b\}$

$= \{a \langle b \langle b\ /_{1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_{k-1}]\ 1\ \langle A_k' \rangle\ b\ (\leftarrow 2)}\ c\ /_{1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_k]\ 2}\ c-1\rangle\ b\rangle\ b\}$,

$\{a, b\ [1\ [1\ [1\ /_{1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_k]\ 3}\ c]\ 2]\ 2]\ 2\}$

$= \{a \langle 0\ [1\ [1\ /_{1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_k]\ 3}\ c]\ 2]\ 2\rangle\ b\}$

$= \{a \langle b \langle b \langle b\ /_{1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_{k-1}]\ 1\ \langle A_k' \rangle\ b\ (\leftarrow 2)\ [A_k]\ 2}\ c\ /_{1\ [A_1]\ 1\ [A_2]\ \ldots\ 1\ [A_k]\ 3}\ c-1\rangle\ b\rangle\ b\rangle\ b\}$,

where A_k' is identical to A_k except that the first entry is reduced by 1.

Taking the arbitrary string S = '1 $[B_1]$ 1 $[B_2]$ ... 1 $[B_m]$ 2', where m ≥ 1 and each $[B_i]$ is a normal separator, the recursive definition of an (n_1 $[A_1]$ n_2 $[A_2]$ ... $[A_k]$ n_{k+1})-hyperseparator (for k ≥ 0, $n_1 \geq 1$, $n_i \geq 1$ (1 ≤ i ≤ k), $n_{k+1} \geq 2$ (k ≥ 1) and normal separators $[A_i]$) is that one of the following four conditions hold:

1. It is the $/_{n_1\ [A_1]\ n_2\ [A_2]\ \ldots\ [A_k]\ n_{k+1}}$ symbol.
2. Contains an (n_1+1 $[A_1]$ n_2 $[A_2]$ n_3 $[A_3]$ ... $[A_k]$ n_{k+1})-hyperseparator as highest hyperseparator in its 'base layer'.
3. Subscripted by 'n_1 $[A_1]$ n_2 $[A_2]$ ... $[A_{i-1}]$ n_i' and contains either a (1 $[A_1]$ 1 $[A_2]$ ... 1 $[A_i]$ n_{i+1}+1 $[A_{i+1}]$ n_{i+2} $[A_{i+2}]$ ... $[A_k]$ n_{k+1})-hyperseparator or (1 $[A_1]$ 1 $[A_2]$ ... 1 $[A_{i-1}]$ S $[A_i]$ n_{i+1} $[A_{i+1}]$ n_{i+2} $[A_{i+2}]$ ... $[A_k]$ n_{k+1})-hyperseparator as highest hyperseparator in its 'base layer', with $[B_1] < [A_i]$, for some 1 ≤ i ≤ k. (When i = k, the hyperseparator expressions would read (1 $[A_1]$ 1 $[A_2]$ ... 1 $[A_k]$ n_{k+1}+1)-hyperseparator or (1 $[A_1]$ 1 $[A_2]$ ... 1 $[A_{k-1}]$ S $[A_k]$ n_{k+1})-hyperseparator.)
4. Subscripted by 'n_1 $[A_1]$ n_2 $[A_2]$ ... $[A_k]$ n_{k+1}' and contains a (1 $[A_1]$ 1 $[A_2]$ ... 1 $[A_k]$ S)-hyperseparator as highest hyperseparator in its 'base layer'.

The highest order hyperseparators in an array are slash symbols, so these are determined first, followed by the next highest hyperseparators, and so on, down to the 1-hyperseparators. A normal separator is a separator that cannot be defined using the above recursive definition – it is neither subscripted nor a slash symbol, and it contains no 2- or higher order hyperseparators in its 'base layer'.

The slash in Rule A5a is now the generalised subscript array $/_M$ symbol, where M is of the form

'm_1 [B_1] m_2 [B_2] ... [B_{q-1}] m_q',

where q ≥ 1, m_q ≥ 2 (if q ≥ 2) and each of [B_j] (1 ≤ j < q) is a normal separator. It works much the same as before when either M = '1' or m_1 ≥ 2 as the $R_{n,s}$ string is the same for all 1 < n < b, only that the [B_j] separators need not be [1]'s (commas). It is now so complex for q ≥ 2, $m_1 = m_2 = ... = m_k = 1$ and m_{k+1} ≥ 2 (for some positive integer k) that the cases [B_k] = [1] (comma) and [B_k] ≥ [2] (non-comma) now have to be treated separately. Hence, Rule A5a now splits up into three subrules:-

Rule A5a: q ≥ 2, m_j = 1 (1 ≤ j ≤ k), m_{k+1} ≥ 2, [B_k] = [1].

Rule A5b: q ≥ 2, m_j = 1 (1 ≤ j ≤ k), m_{k+1} ≥ 2, [B_k] ≥ [2].

Rule A5c: All other scenarios.

In the first two subrules, [B_j] ≥ [B_{j+1}] for all 1 ≤ j < k. The $R_{n,k}$ string nesting operation is only required for subrules a and c; it is not needed for subrule b as a 'shadow' slash subscript array $/_{M^*}$ is created, where M* is identical to M except that the kth and final 1 becomes '1 ‹B_k'› b (←2)' (which resolves to a string of 1's and separators of descending level ending with an entry of 2 after b-1 [1]'s or commas, allowing subrule a to be used for $/_{M^*}$, as b ≥ 2) and the first non-1 entry is reduced by 1 (so that M and M* share the same taxicab distance). Rule A5a now requires a complex algorithm in order to find the correct position within the subscript at the end of the $R_{n,s}$ string (for 1 < n < b, based on that at the end of the [$A_{s-1,p_{s-1}}$] separator) of the entry which increases as n decreases, as the position has to correspond with the kth and final 1 of M – it would be very much more difficult to do this for the new Rule A5b as we would have to resolve both a pair of angle brackets and a left arrow without exiting Rule A5b.

The other subrules of Rule A5, previously known as subrules a*, b and c now become subrules d, e and f respectively. All of the # strings in the Angle Bracket Rules no longer include subscripts at the end of them – these subscripts have now been indicated separately by subscript arrays N (Rules A2-4 and A6) and N_i, $N_{n,k}$, M and M* (Rule A5). The modified and complete Angle Bracket Rules are shown below (the Main or M Rules remain the same as in Nested Array Notation).

Bird's Nested Hierarchical Hyper-Nested Array Notation – Angle Bracket Rules

Rule A1 (only 1 entry of either 0 or 1):

'a ‹0› b' = 'a',
'a ‹1› b' = 'a, a, ... , a' (with b a's),
'a ‹0› b (←c)' = 'c',
'a ‹1› b (←c)' = 'a, a, ... , a, c' (with b-1 a's).

Rule A2 (only 1 entry of either 0 or 1 prior to 2-hyperseparator or higher order hyperseparator):

'a ‹0 # $_N$› b' = 'a',
'a ‹1 # $_N$› b' = 'a [1 # $_N$] a [1 # $_N$] ... [1 # $_N$] a' (with b a's),
'a ‹0 # $_N$› b (←c)' = 'c',
'a ‹1 # $_N$› b (←c)' = 'a [1 # $_N$] a [1 # $_N$] ... a [1 # $_N$] c' (with b-1 a's),

where # begins with a 2- or higher order hyperseparator.

Rule A3 (last entry in any 1-space or higher dimensional space of array is 1):

‘a ‹# [A] 1 $_{N}$› b’ = ‘a ‹# $_{N}$› b’.

When [A] is an M_1-hyperseparator, [B] is an M_2-hyperseparator and $M_1 < M_2$, or $M_1 = M_2$ and [A] < [B],

‘a ‹# [A] 1 [B] #* $_{N}$› b’ = ‘a ‹# [B] #* $_{N}$› b’.

Remove trailing 1’s.

Rule A4 (number to right of angle brackets is 1):

‘a ‹A $_{N}$› 1’ = ‘a’,

‘a ‹A $_{N}$› 1 (←c)’ = ‘c’.

Rule A5 (Rules A1-4 do not apply, first entry is 0, separator immediately prior to next non-1 entry (c_1) is [A_{1,p_1}]):

‘a ‹ 0 [$A_{1,1}$] 1 [$A_{1,2}$] ... 1 [A_{1,p_1}] c_1 $\#_1$ #* $_{N_1}$› b’ = ‘a ‹S_1 #* $_{N_1}$› b’,

where $p_1 \geq 1$, each of [$A_{1,j}$] is either a normal separator or 1-hyperseparator, $\#_1$ contains no 2- or higher order hyperseparators in its base layer and #* is either an empty string or begins with a 2- or higher order hyperseparator.

Set i to 1 and t_1 to 0 and follow Rules A5a-f (a-d and f are terminal, e is not). (Note that $i = t_1+1$ throughout.)

Rule A5a (separator [A_{i,p_i}] = $/_M$,

where M = ‘1 [B_1] 1 [B_2] ... [B_{p-1}] 1, m_1 [C_1] m_2 [C_2] ... [C_{q-1}] m_q’,

where $p \geq 1$, $q \geq 1$, $m_1 \geq 2$, $m_q \geq 2$ and each of [B_j] ($1 \leq j < p$) and [C_j] ($1 \leq j < q$) are normal separators):

Set r to 1. For each j from 1 to i-1, increment r by 1 when $M_j < M$.

$s = i-t_r$.

The subscript array N_s = ‘n_1 [D_1] n_2 [D_2] ... [D_{h-1}] n_h’,

where $h \geq 1$, $n_h \geq 2$ ($h \geq 2$) and each of [D_j] ($1 \leq j < h$) is a normal separator.

Follow 3-step algorithm (below) until the $N_{n,s}$ string (for $1 < n < b$) is determined.

Step 1: Set x and y to 1 and go to Step 2.

Step 2: If x = p and y = h then $N_{n,s}$ = ‘n_1 [D_1] n_2 [D_2] ... n_{h-1} [D_{h-1}] n_h+b-n’.

If x = p and y < h then $N_{n,s}$ = ‘n_1 [D_1] n_2 [D_2] ... n_{y-1} [D_{y-1}] n_y+b-n [D_y] n_{y+1} [D_{y+1}] ... n_{h-1} [D_{h-1}] n_h’.

If x < p and y = h then $N_{n,s}$ = ‘n_1 [D_1] n_2 [D_2] ... n_{h-1} [D_{h-1}] n_h [B_x] 1 [B_{x+1}] ... 1 [B_{p-1}] b+1-n’.

If x < p and y < h then go to Step 3.

Step 3: If [B_x] > [D_y] then increment y by 1 and go to Step 2.

If [B_x] = [D_y] then increment x and y by 1 and go to Step 2.

If [B_x] < [D_y] then $N_{n,s}$ = ‘n_1 [D_1] n_2 [D_2] ... n_{y-1} [D_{y-1}] n_y [B_x] 1 [B_{x+1}] ... 1 [B_{p-1}] b+1-n [D_y] n_{y+1} [D_{y+1}] ... n_{h-1} [D_{h-1}] n_h’.

We then obtain

$N_{b,i}$ = ‘N_i’,

$N_{n,k}$ = ‘N_k’ ($1 < n < b$, $s < k \leq i$),

S_i = ‘$R_{b,i}$’.

For $1 < n \leq b$ and $s \leq k < i$,

$R_{n,i}$ = ‘b ‹$A_{i,1}$’› b [$A_{i,1}$] b ‹$A_{i,2}$’› b [$A_{i,2}$] ... b ‹A_{i,p_i-1}’› b [A_{i,p_i-1}] b ‹$R_{n-1,s}$› b $/_M$ c_i-1 $\#_i$ $_{N_{n,i}}$’,

$R_{n,k}$ = ‘b ‹$A_{k,1}$’› b [$A_{k,1}$] b ‹$A_{k,2}$’› b [$A_{k,2}$] ... b ‹A_{k,p_k-1}’› b [A_{k,p_k-1}] b ‹$R_{n,k+1}$› b [A_{k,p_k}] c_k-1 $\#_k$ $_{N_{n,k}}$’,

$R_{1,s}$ = ‘0’.

Rule A5b (Rule A5a does not apply, separator $[A_{i,p_i}] = /_M$,
where M = ‘$1\ [B_1]\ 1\ [B_2]\ ...\ [B_{p-1}]\ 1\ [B_p]\ m_1\ [C_1]\ m_2\ [C_2]\ ...\ [C_{q-1}]\ m_q$’,
where $p \geq 1$, $q \geq 1$, $m_1 \geq 2$, $m_q \geq 2$, $[B_p] \geq [2]$ and each of $[B_j]$ ($1 \leq j < p$) and $[C_j]$ ($1 \leq j < q$) are normal separators):

S_i = ‘$b\ ‹A_{i,1}'›\ b\ [A_{i,1}]\ b\ ‹A_{i,2}'›\ b\ [A_{i,2}]\ ...\ b\ ‹A_{i,p_i-1}'›\ b\ [A_{i,p_i-1}]\ b\ \ /_{M^*}\ \ c_i\ \#_i\ \ /_M\ \ c_i\text{-}1\ \#_i\ _{N_i}$’,
where M^* = ‘$1\ [B_1]\ 1\ [B_2]\ ...\ [B_{p-1}]\ 1\ ‹B_p'›\ b\ (\leftarrow 2)\ [B_p]\ m_1\text{-}1\ [C_1]\ m_2\ [C_2]\ ...\ [C_{q-1}]\ m_q$’.

Rule A5c (Rules A5a-b do not apply, separator $[A_{i,p_i}] = /_M$,
where M = ‘$m_1\ [B_1]\ m_2\ [B_2]\ ...\ [B_{q-1}]\ m_q$’,
where $q \geq 1$, $m_1 \geq 2$ ($q \geq 2$), $m_q \geq 2$ ($q \geq 2$) and each of $[B_j]$ ($1 \leq j < q$) is a normal separator):

Set r to 1. For each j from 1 to i-1, increment r by 1 when $M_j < M$.
$s = i\text{-}t_r$,
S_i = ‘$R_{b,i}$’.
For $1 < n \leq b$ and $s \leq k < i$,
$R_{n,i}$ = ‘$b\ ‹A_{i,1}'›\ b\ [A_{i,1}]\ b\ ‹A_{i,2}'›\ b\ [A_{i,2}]\ ...\ b\ ‹A_{i,p_i-1}'›\ b\ [A_{i,p_i-1}]\ b\ ‹R_{n-1,s}›\ b\ \ /_M\ \ c_i\text{-}1\ \#_i\ _{N_i}$’,
$R_{n,k}$ = ‘$b\ ‹A_{k,1}'›\ b\ [A_{k,1}]\ b\ ‹A_{k,2}'›\ b\ [A_{k,2}]\ ...\ b\ ‹A_{k,p_k-1}'›\ b\ [A_{k,p_k-1}]\ b\ ‹R_{n,k+1}›\ b\ [A_{k,p_k}]\ c_k\text{-}1\ \#_k\ _{N_k}$’,
$R_{1,s}$ = ‘0’.

Rule A5d (Rules A5a-c do not apply, separator $[A_{i,p_i}] = [d\ \#_H\ _M]$, where $d \geq 2$ and $\#_H$ contains a H-hyperseparator as the highest order hyperseparator in its base layer,
where H = ‘$1\ [H_1]\ 1\ [H_2]\ ...\ 1\ [H_k]\ h\ \#_{H^*}$’,
where $h \geq 2$, $k \geq 1$ and each of $[H_j]$ ($1 \leq j \leq k$) is a normal separator):

S_i = ‘$b\ ‹A_{i,1}'›\ b\ [A_{i,1}]\ b\ ‹A_{i,2}'›\ b\ [A_{i,2}]\ ...\ b\ ‹A_{i,p_i-1}'›\ b\ [A_{i,p_i-1}]\ \ R_b\ \ [d\ \#_H\ _M]\ \ c_i\text{-}1\ \#_i\ _{N_i}$’,
R_n = ‘$b\ ‹R_{n-1}›\ b$’ ($n > 1$),
R_1 = ‘$b\ \ [d\text{-}1\ \#_{H\ 1\ [H_1]\ 1\ [H_2]\ ...\ 1\ [H_{k-1}]\ 1\ ‹H_k'›\ b\ (\leftarrow m+b-1)}]\ \ b$’,
where m (which may be 1) is the kth and final entry in the subscript array M when written as
M = ‘$m_1\ [H_1]\ m_2\ [H_2]\ ...\ m_{k-1}\ [H_{k-1}]\ m$’.

Rule A5e (Rules A5a-d do not apply, separator $[A_{i,p_i}] = [1\ [A_{i+1,1}]\ 1\ [A_{i+1,2}]\ ...\ 1\ [A_{i+1,p_{i+1}}]\ c_{i+1}\ \#_{i+1}\ _{N_{i+1}}]$, which is an M_i-hyperseparator, where $p_{i+1} \geq 1$, $c_{i+1} \geq 2$ and $M_i \geq$ ‘1’):

S_i = ‘$b\ ‹A_{i,1}'›\ b\ [A_{i,1}]\ b\ ‹A_{i,2}'›\ b\ [A_{i,2}]\ ...\ b\ ‹A_{i,p_i-1}'›\ b\ [A_{i,p_i-1}]\ b\ ‹S_{i+1}›\ b\ [A_{i,p_i}]\ c_i\text{-}1\ \#_i\ _{N_i}$’.
Set r to 1 and f to 0. For each j from 1 to i-1, increment r by 1 when $M_j \leq M_i$ and set f to 1 when $M_j = M_i$.
Increment $t_1, t_2, ... , t_r$ by 1; set t_r to t_{r-1} if f = 1; reset $t_{r+1}, t_{r+2}, ... , t_i$ to 0; set t_{i+1} to 0; increment i by 1 (so that $i = t_1+1$) and repeat Rules A5a-f.

Rule A5f (Rules A5a-e do not apply):

S_i = ‘$b\ ‹A_{i,1}'›\ b\ [A_{i,1}]\ b\ ‹A_{i,2}'›\ b\ [A_{i,2}]\ ...\ b\ ‹A_{i,p_i}'›\ b\ [A_{i,p_i}]\ c_i\text{-}1\ \#_i\ _{N_i}$’.

Rule A6 (Rules A1-5 do not apply):

‘$a\ ‹n\ \#\ _N›\ b$’ = ‘$a\ ‹n\text{-}1\ \#\ _N›\ b\ [n\ \#\ _N]\ a\ ‹n\text{-}1\ \#\ _N›\ b\ [n\ \#\ _N]\ ...\ [n\ \#\ _N]\ a\ ‹n\text{-}1\ \#\ _N›\ b$’
(with b ‘$a\ ‹n\text{-}1\ \#\ _N›\ b$’ strings).

Notes:

1. A, B, $A_{i,1}$, $A_{i,2}$, ... , A_{i,p_i}, B_j, C_j, D_j and H_j are strings of characters within separators. In Rules A3 and A5, the strings A, B, $A_{i,1}$, $A_{i,2}$, ... , A_{i,p_i} ($i \geq 2$) may include subscripts at the end.

2. $A_{i,1}$', $A_{i,2}$', ... , A_{i,p_i}', B_p' and H_k' are strings of characters within angle brackets that are identical to the strings $A_{i,1}$, $A_{i,2}$, ... , A_{i,p_i}, B_p and H_k respectively except that the first entries of each have been reduced by 1. If $A_{i,j}$ (for some $1 \le j \le p_i$) begins with 1, $A_{i,j}$' begins with 0.
3. M, M*, N, N_i and $N_{n,k}$ are strings of characters that make up subscript arrays.
4. M_i and H are strings of characters that describe hyperlevels (hyperseparator levels).
5. S_i, R_n and $R_{n,k}$ are string building functions which create strings of characters. The R functions involve nesting the same string of characters around itself n-1 times before being replaced by the string '0'.
6. #, #*, $\#_i$, $\#_H$ and $\#_{H^*}$ are strings of characters representing the remainder of the array (can be null or empty), excluding any subscripts at the end. Here, the H and H* subscripts are labels and not variables or strings of characters.
7. In Rules A5b and A5d, a 'left arrow' is created, which is shown within brackets next to a number and placed to the right of an angle bracket array (with 1 and b either side), namely '1 $\langle B_p' \rangle$ b (←2)' within M* in the former subrule and '1 $\langle H_k' \rangle$ b (←m+b-1)' within the subscript array within R_1 in the latter. The angle brackets resolve to a string of 1's and separators (of descending level with b-1 of each level down to the [1] separator or comma), and the final entry of 1 is replaced by the number next to the 'left arrow'. The arrow is finally removed along with the angle brackets immediately to its left, using Rules A1, A2 or A4.
8. A $/_N$ symbol is an N-hyperseparator. A recursive definition of hyperseparators is given on page 41. A separator that contains no 2- or higher order hyperseparators in its 'base layer' is a normal separator (or 0-hyperseparator).
9. The comma is used as shorthand for the [1] separator.
10. When a separator [A] = $/_N$ and it is necessary to find the string A' (identical to A but for the first entry being reduced by 1), the $/_N$ is used as shorthand for the [1 $/_{N^*}$ 2] separator, where N* is identical to N except that the first entry has been increased by 1. In this case, A' = '0 $/_{N^*}$ 2', which means that 'b $\langle$A'$\rangle$ b' = 'b $\langle 0 /_{N^*} 2 \rangle$ b' = 'b' (by Rule A2).
11. Any 2- or higher hyperseparator may carry a subscript.
12. The [1 $/_M$ 2 $_N$] separator (M ≠ '1') reduces to a $/_X$ symbol, for a subscript array X, according to a special rule (see below).

Whenever we encounter a separator of the form [1 $/_X$ 2 $_Y$] (with a slash symbol between the only two entries of 1 and 2), where X and Y are subscript arrays and X ≠ '1', this separator 'drops down' to a simple slash symbol of the form $/_Z$, where Z is another subscript array. This special rule is known as the Dropping Down Rule.

Dropping Down Rule (separator is of the form [1 $/_M$ 2 $_N$] with M ≠ '1', i.e. has only two entries of 1 and 2 to left and right respectively of slash subscript array, which is a 2- or higher order hyperseparator):

$$[1 \ /_{1\,[A_1]\,1\,[A_2]\,\ldots\,1\,[A_m]\,n_{m+1}\,\#} \ 2 \ _{n_1\,[A_1]\,n_2\,[A_2]\,\ldots\,[A_{k-1}]\,n_k}]$$
$$= /_{n_1\,[A_1]\,n_2\,[A_2]\,\ldots\,n_k\,[A_k]\,1\,[A_{k+1}]\,1\,[A_{k+2}]\,\ldots\,1\,[A_m]\,n_{m+1}-1\,\#},$$

where $0 \le k \le m$, $n_{m+1} \ge 2$, each of $[A_i]$ is a normal separator and # represents the remainder of the slash subscript array.

When k = m, the Dropping Down Rule becomes:

$$[1 \ /_{1\,[A_1]\,1\,[A_2]\,\ldots\,1\,[A_m]\,n_{m+1}\,\#} \ 2 \ _{n_1\,[A_1]\,n_2\,[A_2]\,\ldots\,[A_{m-1}]\,n_m}]$$
$$= /_{n_1\,[A_1]\,n_2\,[A_2]\,\ldots\,n_m\,[A_m]\,n_{m+1}-1\,\#}.$$

When k = 0, the Dropping Down Rule becomes:

$$[1 \ /_{1\,[A_1]\,1\,[A_2]\,\ldots\,1\,[A_m]\,n_{m+1}\,\#} \ 2]$$
$$= /_{1\,[A_1]\,1\,[A_2]\,\ldots\,1\,[A_m]\,n_{m+1}-1\,\#}.$$

When k = m = 0, the Dropping Down Rule becomes:

$[1 /_{n_1 \#} 2] = /_{n_1\text{-}1 \#}$.

Trailing 1's in subscript arrays are removed as with separator arrays and angle bracket arrays. For example,

$/_{\# [A] 1} = /_{\#}$ and $[X_{\# [A] 1}] = [X_{\#}]$.

When [A] < [B],

$/_{\# [A] 1 [B] \#*} = /_{\# [B] \#*}$ and $[X_{\# [A] 1 [B] \#*}] = [X_{\# [B] \#*}]$.

This is my complete Nested Hierarchical Hyper-Nested Array Notation. The limit ordinal of this notation is $\theta(\Omega_\Omega)$. The Extended Kruskal Theorem (Kruskal's Tree Theorem extended to labelled trees) and Buchholz Hydras with finite numbers as labels gets us as far as the $\theta(\Omega_\omega)$ level in the fast-growing hierarchy. However, if we use trees as labels for trees, then use those trees as labels for even larger trees, and so on, we also reach a limit of $\theta(\Omega_\Omega)$ in the hierarchy of fast-growing functions!

This notation works for simple nested arrays up to ε_0 level without requiring Rules A2 and A5a-e. These rules only become necessary when one wishes to travel beyond the following ordinals or use the associated separators, as shown below:

Ordinal	**Separator**	**Rule**
ε_0	[1 / 2]	A5c
$\varphi(\omega, 0)$	$[1\ [2\ /_2\ 2]\ 2]$	A2
Γ_0	$[1\ [1\ /\ 2\ /_2\ 2]\ 2]$	A5e
$\theta(\Omega_\omega)$	$[1\ [2\ /_{1,2}\ 2]\ 2]$	A5d
$\theta(\Omega_\omega+1)$	$[1\ [1\ /_{1,2}\ 3]\ 2]$	A5a
$\theta(\Omega_{\omega\text{^}\omega}+1)$	$[1\ [1\ /_{1\ [2]\ 2}\ 3]\ 2]$	A5b

In Rule A5a, the 3-step algorithm begins initially with Step 1 (set x and y to 1), then shuttles between the other two steps until a solution for the $N_{n,s}$ subscript array is found. In Step 2, $N_{n,s}$ is determined when either x = p or y = h (reached end of either $[D_j]$ separators or $[B_j]$ preceding final 1 and comma prior to first non-1 entry), otherwise it moves to Step 3. In Step 3, $N_{n,s}$ is immediately found when the xth B-separator of M is of lower level than the yth D-separator of N_s, or $[B_x] < [D_y]$ (as $[B_j] < [D_y]$ for all x < j < p), otherwise y is increased by 1 (and also x if $[B_x]$ and $[D_y]$ are identical) before jumping back to Step 2.

Two simple examples using the 3-step algorithm suffice for given subscript arrays of

M = '1 $[B_1]$ 1 $[B_2]$ ... $[B_{p-1}]$ 1, m_1 $[C_1]$ m_2 $[C_2]$ ... $[C_{q-1}]$ m_q',

N_s = 'n_1 $[D_1]$ n_2 $[D_2]$ ... $[D_{h-1}]$ n_h'.

Example 1:

M = '1 [4] 1 [2] 1, 4, 4 [5] 2' (p = 3),

N_s = '1 [3] 2 [4] 3, 4 [2] 5, 6 [3] 7' (h = 7).

x = 1, y = 1: As $[B_x] > [D_y]$ ([4] > [3]), increment y by 1.

x = 1, y = 2: As $[B_x] = [D_y]$ ([4] = [4]), increment x and y by 1.

x = 2, y = 3: As $[B_x] > [D_y]$ ([2] > [1]), increment y by 1.

x = 2, y = 4: As $[B_x] = [D_y]$ ([2] = [2]), increment x and y by 1.

x = 3, y = 5: As x = p and y < h, we obtain the subscript array

$N_{n,s}$ = 'n_1 $[D_1]$ n_2 $[D_2]$ n_3 $[D_3]$ n_4 $[D_4]$ n_5+b-n $[D_5]$ n_6 $[D_6]$ n_7'

= '1 [3] 2 [4] 3, 4 [2] b+5-n, 6 [3] 7'.

Example 2:

M = ‘1 [5] 1 [5] 1 [3] 1 [3] 1 [2] 1, 7, 7 [8] 2’ (p = 6),

N_s = ‘1 [5] 2 [3] 3 [5] 4 [4] 5, 6 [6] 7’ (h = 7).

x = 1, y = 1: As $[B_x] = [D_y]$ ([5] = [5]), increment x and y by 1.

x = 2, y = 2: As $[B_x] > [D_y]$ ([5] > [3]), increment y by 1.

x = 2, y = 3: As $[B_x] = [D_y]$ ([5] = [5]), increment x and y by 1.

x = 3, y = 4: As $[B_x] < [D_y]$ ([3] < [4]), we obtain the subscript array

$N_{n,s}$ = ‘$n_1\ [D_1]\ n_2\ [D_2]\ n_3\ [D_3]\ n_4\ [B_3]\ 1\ [B_4]\ 1\ [B_5]$ b+1-n $[D_4]\ n_5\ [D_5]\ n_6\ [D_6]\ n_7$’

= ‘1 [5] 2 [3] 3 [5] 4 [3] 1 [3] 1 [2] b+1-n [4] 5, 6 [6] 7’.

Rule A5a with every $[B_j]$ and $[D_j]$ equal to [1] (comma) would always have x = y in the 3-step algorithm and would achieve the following results for the $N_{n,s}$ subscript array (1 < n < b):

$N_{n,s}$ = ‘$n_1, n_2, \ldots, n_{h-1}, n_h$+b-n’ (p = h)

= ‘$n_1, n_2, \ldots, n_{p-1}, n_p$+b-n, $n_{y+1}, \ldots, n_h$’ (p < h)

= ‘$n_1, n_2, \ldots, n_h$, 1, ... , 1, b+1-n’ (with p-h-1 1’s) (p > h).

Rule A5b involves neither an algorithm to calculate $N_{n,s}$ subscript arrays nor an $R_{n,k}$ string nesting operation for reasons explained on page 42. Only the S_i string is determined, which creates the string ‘b $/_{M^*}$ c_i $\#_i$’ in the space between the separators $[A_{i,p_i-1}]$ and $/_M$. M* is a ‘shadow’ subscript array which ‘tends to’ M as b tends to infinity. The entry to the left of $/_{M^*}$ is b as this is ‘b ‹0 $/_{M^{**}}$ 2› b’, where M** is identical to M* apart from the addition of 1 to the first entry. Simple examples of arrays that use this rule are shown on page 41.

Rule A5c is similar to Rule A5a except that M is either a single entry of ‘1’ or begins with an entry of at least 2 and there is no algorithm to calculate $N_{n,s}$ subscript arrays since $N_{n,s}$ would be identical to N_s for all 1 < n ≤ b. After finding the ‘rank’ of M among the M_j strings from M_1 to M_{i-1} (final value of r) and setting $s = i-t_r$ and S_i = ‘$R_{b,i}$’, we jump straight to determining the $R_{n,k}$ strings in the order shown as follows:

$R_{b,i}, R_{b-1,s}, R_{b-1,s+1}, \ldots, R_{b-1,i}, R_{b-2,s}, R_{b-2,s+1}, \ldots, R_{b-2,i}, \ldots\ldots, R_{2,s}, R_{2,s+1}, \ldots, R_{2,i}, R_{1,s}$.

Each of the $R_{n,k}$ strings (for 1 < n ≤ b and s ≤ k ≤ i) ends with the N_k subscript array and $R_{1,s}$ = ‘0’.

Rule A5d with the lowest H-hyperseparator within $\#_H$,

$[A_{i,p_i}] = [2\ /_{1\ [H_1]\ 1\ [H_2]\ \ldots\ 1\ [H_k]\ h\ \#_{H^*}}\ 2\ _M]$

would mean that

R_1 = ‘b $[1\ /_{1\ [H_1]\ 1\ [H_2]\ \ldots\ 1\ [H_k]\ h\ \#_{H^*}}\ 2\ _{1\ [H_1]\ 1\ [H_2]\ \ldots\ 1\ [H_{k-1}]\ 1\ \langle H_k' \rangle\ b\ (\leftarrow m+b-1)}]$ b’

= ‘b $/_{1\ [H_1]\ 1\ [H_2]\ \ldots\ 1\ [H_{k-1}]\ 1\ \langle H_k' \rangle\ b\ (\leftarrow m+b-1)\ [H_k]\ h-1\ \#_{H^*}}$ b’ (Dropping Down Rule),

which is how the (k+1)th entry of the slash subscript array is reduced by 1. (The kth entry is completely filled up with 1’s in the space corresponding to the separator $[H_k]$, with b 1’s in each ‘row’, b ‘rows’ in each ‘plane’, etc., and the very last of these 1’s is replaced by m+b-1; all other entries remain unchanged.) When every $[H_j]$ is equal to [1] (comma), we would have

$[A_{i,p_i}] = [2\ /_{1,1,\ldots,1,h\ \#_{H^*}}\ 2\ _M]$ (with k 1’s)

and R_1 = ‘b $/_{1,\ldots,1,m+b-1,h-1\ \#_{H^*}}$ b’ (with k-1 1’s; kth entry is ‘1 ‹0› b (←m+b-1)’ = ‘m+b-1’),

where m (which may be 1) is the kth and final entry in the subscript array M when written as

M = ‘$m_1,m_2,\ldots,m_{k-1},m$’.

This is similar to the old Rule A5a* for linear subscript arrays (see page 36) but with ‘h $\#_{H^*}$’ representing ‘$r_1, r_2, \ldots$’ and m representing m_k.

In Rule A5e, M_1 = ‘1’ since [A_{1,p_1}] is always a 1-hyperseparator. M_2 (hyperlevel of [A_{2,p_2}]) can be either ‘1’, ‘2’ or ‘1 [X_1] 1 [X_2] ... 1 [X_n] 2’. The possible values of M_3 (hyperlevel of [A_{3,p_3}]) are dependent on the value of M_2 as follows:

‘1’ (any value of M_2),
‘2’ (any value of M_2),
‘3’ (M_2 = ‘2’),
‘1 [X_1] 1 [X_2] ... 1 [X_n] 2’ (any value of M_2),
‘1 [Y_1] 1 [Y_2] ... 1 [Y_m] 2 [X_{k+1}] 1 [X_{k+2}] ... 1 [X_n] 2’ (M_2 = ‘1 [X_1] 1 [X_2] ... 1 [X_n] 2’),
‘1 [X_1] 1 [X_2] ... 1 [X_n] 3’ (M_2 = ‘1 [X_1] 1 [X_2] ... 1 [X_n] 2’),

with 1’s making up the remaining entries, $n \geq 1$, $m \geq k \geq 0$ and

‘1 [Y_1] 1 [Y_2] ... 1 [Y_m] 1 [X_{k+1}] 1 [X_{k+2}] ... 1 [X_n] 2’ = ‘1 [X_1] 1 [X_2] ... 1 [X_n] 2’.

When M_k (hyperlevel of [A_{k,p_k}]) is taken to be

‘m_1 [B_1] m_2 [B_2] ... [B_{q-1}] m_q’,

travelling through the M_i sequence in reverse order from M_k to M_1 and taking note of those of lower values entails passing through all of the following values of M_i:

‘j [B_1] m_2 [B_2] ... [B_{q-1}] m_q’ (from j = m_1-1 down to j = 1),
‘1 [B_1] j [B_2] m_3 [B_3] ... [B_{q-1}] m_q’ (from j = m_2-1 down to j = 1),
‘1 [B_1] 1 [B_2] j [B_3] m_4 [B_4] ... [B_{q-1}] m_q’ (from j = m_3-1 down to j = 1),
......
‘1 [B_1] 1 [B_2] ... 1 [B_{q-1}] j’ (from j = m_q-1 down to j = 1).

Can anyone beat this function with growth rate at the $\theta(\Omega_\Omega)$ level? It is defined as follows:

S(n) = {3, n [1 [2 /$_{R_n}$ 2] 2] 2},

where R_i = ‘1 [1 [2 /$_{R_{i-1}}$ 2] 2] 2’,
R_1 = ‘1, 2’.

While S(1) = 3, the number

S(2) = {3, 2 [1 [2 /$_{1\ [1\ [2\ /_{1,2}\ 2]\ 2]\ 2}$ 2] 2] 2}
= {3 ‹0 [2 /$_{1\ [1\ [2\ /_{1,2}\ 2]\ 2]\ 2}$ 2] 2› 2}
= {3 ‹2 ‹2 /$_A$ 2› 2› 2},

where A = ‘1 ‹0 [2 /$_{1,2}$ 2] 2› 2 (←2)’
= ‘1 ‹2 ‹2 /$_2$ 2› 2› 2 (←2)’
= ‘1 ‹2 / 2 [2 /$_2$ 2] 2 / 2› 2 (←2)’
= ‘1 [2/2 [2 /$_2$ 2] 2/2] 1 [1/2 [2 /$_2$ 2] 2/2] 1 [2 [2 /$_2$ 2] 2/2] 1 [1 [2 /$_2$ 2] 2/2]
1 [2/2 [2 /$_2$ 2] 1/2] 1 [1/2 [2 /$_2$ 2] 1/2] 1 [2 [2 /$_2$ 2] 1/2] 1 [1 [2 /$_2$ 2] 1/2]
1 [2/2 [2 /$_2$ 2] 2] 1 [1/2 [2 /$_2$ 2] 2] 1 [2 [2 /$_2$ 2] 2] 1 [1 [2 /$_2$ 2] 2] 1 [2/2] 1 [1/2] 1 [2] 1,2’,

which means that

S(2) = {3 ‹2 [1 /$_A$ 2] 2 [2 /$_A$ 2] 2 [1 /$_A$ 2] 2› 2}
= {3 ‹2 / 2 [2 /$_A$ 2] 2 / 2› 2} ([1 /$_A$ 2] = / as A has single non-1 entry of 2)
= {B [1/2 [2 /$_A$ 2] 2/2] B [2/2 [2 /$_A$ 2] 2/2] B [1/2 [2 /$_A$ 2] 2/2] B},

where B = ‘3 ‹0 / 2 [2 /$_A$ 2] 2 / 2› 2’
= ‘3 ‹2 [2 /$_A$ 2] 2 / 2› 2’
= ‘C [1 [2 /$_A$ 2] 2/2] C [2 [2 /$_A$ 2] 2/2] C [1 [2 /$_A$ 2] 2/2] C’,

where C = ‘3 ‹0 [2 /$_A$ 2] 2 / 2› 2’
= ‘3 ‹2 ‹2 /$_{A^*}$ 2› 2 [2 /$_A$ 2] 1 / 2› 2’,

where A* is identical to A except that the final ‘1,2’ is replaced by ‘1 ‹0› 2 (←2)’ = ‘2’ (Rule A5d), which means that

C = ‘3 ‹2 [1 /$_{A^*}$ 2] 2 [2 /$_{A^*}$ 2] 2 [1 /$_{A^*}$ 2] 2 [2 /$_A$ 2] 1 / 2› 2’

= ‘$3 \langle 2 / 2 [2 /_{A*} 2] 2 / 2 [2 /_A 2] 1 / 2 \rangle 2$’ ($[1 /_{A*} 2] = /$ as A* has single non-1 entry).

It would be rather tedious to go any further – we would eventually encounter a string where there are 65,536 $[2 /_{A_i} 2]$ separators in

‘$3 \langle 2 / 2 [2 /_2 2] 2 / 2 [2 /_{1,2} 2] 1 / 2 [2 /_{1\,[2]\,2} 2] 1 / 2 \ldots [2 /_{A*} 2] 1 / 2 [2 /_A 2] 1 / 2 \rangle 2$’,

where each A_{i+1} is identical to A_i (starting with $A_1 = A$ and $A_2 = A*$) except that the ‘$1 [X_{i+1}] 2$’ at the end of A_{i+1} (taking $[X_{i+1}]$ as its final separator) is replaced by ‘$1 \langle X_{i+1}\text{’} \rangle 2 (\leftarrow 2)$’ (Rule A5d), before the last A_i contains a single entry.

The third value of the S function,

$S(3) = \{3, 3\ [1 [2 /_{1\,[1\,[2\,/_A\,2]\,2]\,2} 2] 2]\ 2\}$ (with A = ‘$1 [1 [2 /_{1,2} 2] 2] 2$’)

$= \{3 \langle 0 [2 /_{1\,[1\,[2\,/_A\,2]\,2]\,2} 2] 2 \rangle 3\}$

$= \{3 \langle 3 \langle 3 \langle 3 /_B 3 \rangle 3 \rangle 3 \rangle 3\}$,

where B = ‘$1 \langle 0 [2 /_A 2] 2 \rangle 3 (\leftarrow 3)$’

= ‘$1 \langle 0 [2 /_{1\,[1\,[2\,/_{1,2}\,2]\,2]\,2} 2] 2 \rangle 3 (\leftarrow 3)$’

= ‘$1 \langle 3 \langle 3 \langle 3 /_{1\,\langle 0\,[2\,/_{1,2}\,2]\,2 \rangle\,3\,(\leftarrow 3)} 3 \rangle 3 \rangle 3 \rangle 3 (\leftarrow 3)$’

= ‘$1 \langle 3 \langle 3 \langle 3 /_{1\,\langle 3\,\langle 3\,\langle 3\,/_3\,3 \rangle\,3 \rangle\,3 \rangle\,3\,(\leftarrow 3)} 3 \rangle 3 \rangle 3 \rangle 3 (\leftarrow 3)$’.

In general,

$S(n) = \{3 \langle n \langle n \langle \ldots \langle n \langle n /_{R_n} n \rangle n \rangle \ldots \rangle n \rangle n \rangle n\}$ (with n pairs of angle brackets),

where R_i = ‘$1 \langle n \langle n \langle \ldots \langle n \langle n /_{R_{i-1}} n \rangle n \rangle \ldots \rangle n \rangle n \rangle n (\leftarrow n)$’ (with n pairs of angle brackets),

R_1 = ‘n’.

Imagine how huge this number must be:

$S(S(S(\ldots S(3)\ldots)))$ (with S(3) S’s).

It surely must be scraping infinity!

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